ταξίδι στο τοί700d

a)

$$x(t) = \sqrt{\frac{c^4}{a_0^2} + (ct)^2} - \frac{c^2}{a_0} \rightarrow \left(x + \frac{c^2}{a_0}\right)^2 - ct^2 = \frac{c^4}{a_0^2}$$

↑ υπερβολική κίνηση

$$t = \frac{1}{c}\sqrt{\left(x + \frac{c^2}{a_0}\right)^2 - \frac{c^4}{a_0^2}}$$

$a_0 = 9.81\ \frac{m}{s^2}$

για $x = 100$ ly $\simeq 9.46 \cdot 10^7$ m $t \simeq 101$ y

$\gamma d\tau = dt$

$$dx/dt = \frac{c^2 t}{\sqrt{c^2t^2 + c^4/a_0^2}} = u(t)$$

B)

$$\gamma^{-2} = 1 - (u/c)^2 = \frac{c^4/a_0^2}{c^2t^2 + c^4/a_0^2} \qquad \tau = \int_0^t \sqrt{1 - \left(\frac{u}{c}\right)^2}\, dt'$$

$$\tau = \int_0^t \sqrt{\frac{c^4/a_0^2}{c^2t'^2 + c^4/a_0^2}}\, dt' = \int_0^t \frac{1}{\sqrt{\left(\frac{a_0 t'}{c}\right)^2 + 1}}\, dt' = \frac{c}{a}\, \text{arsinh}\left(\frac{at}{c}\right)$$

για $t \simeq 101$ y $\tau \simeq 5.2$ y

γ)

$$u^\mu = (c\gamma, \gamma u) = \left(c\sqrt{\frac{c^2t^2 + c^4/a_0^2}{c^4/a_0^2}},\ a_0 t\right) = \left(c\sqrt{\left(\frac{a_0 t}{c}\right)^2 + 1},\ a_0 t\right)$$

$$a^\mu = \gamma\left(\frac{a_0^2 t}{c\sqrt{\left(\frac{a_0 t}{c}\right)^2 + 1}},\ a_0 t\right) = \left(\frac{a_0^2 t}{c},\ a_0\sqrt{\left(\frac{a_0 t}{c}\right)^2 + 1}\right)$$

$$a^\mu a_\mu = -\left(\frac{a_0^2}{c} t\right)^2 + a_0^2\left(\frac{a_0 t}{c}\right)^2 + a_0^2 = a_0^2$$

δ)

$$\frac{dp}{dt} = ma_0 = \frac{d}{dt}(\gamma m u) \qquad \frac{d}{dt}(\gamma u) = a_0 \qquad \gamma u = a_0 t$$

$$u = \frac{a_0 t}{\sqrt{1 + (a_0 t/c)^2}} = \frac{dx}{dt} \qquad x = \int \frac{a_0 t}{\sqrt{1 + (a_0 t/c)^2}}\, dt \quad \text{με } x(0) = 0$$

$$x(t) = \sqrt{\frac{c^4}{a_0^2} + (ct)^2} - \frac{c^2}{a_0}$$

https://www.desmos.com/calculator/cjc1pw8yce