$\boxed{\Lambda^\mu{}_\kappa \Lambda_\mu{}^\nu = \delta^\nu_\kappa}$ ────────────────────

a)

Θελουμε $\qquad A'^\mu A'_\mu = A^\mu A_\mu$

οπου $\qquad A^\mu \rightarrow A'^\mu = \Lambda^\mu{}_\nu A^\nu$

$\qquad\qquad A_\mu \rightarrow A'_\mu = \Lambda_\mu{}^\nu A_\nu = \Lambda_\mu{}^\kappa A_\kappa$

$A'^\mu A'_\mu = \Lambda^\mu{}_\nu A^\nu \Lambda_\mu{}^\kappa A_\kappa = \Lambda^\mu{}_\nu \Lambda_\mu{}^\kappa A^\nu A_\kappa$

αρα πρεπει $\qquad \boxed{\Lambda^\mu{}_\nu \Lambda_\mu{}^\kappa = \delta^\kappa_\nu}$ ωστε να ισχυει

$A'^\mu A'_\mu = \Lambda^\mu{}_\nu \Lambda_\mu{}^\kappa A^\nu A_\kappa = A^\nu A_\nu = A^\mu A_\mu$

b)

$A'^\mu = \Lambda^\mu{}_\nu A^\nu$

$A'_\mu = \eta_{\mu\kappa} A'^\kappa = \eta_{\mu\kappa} \Lambda^\kappa{}_\sigma A^\sigma = \eta_{\mu\kappa} \Lambda^\kappa{}_\sigma \eta^{\sigma\nu} A_\nu$

αρα πρεπει $\qquad \boxed{\Lambda_\mu{}^\nu = \eta_{\mu\kappa} \Lambda^\kappa{}_\sigma \eta^{\sigma\nu}}$

ωστε $\qquad A'^\mu \cdot A'_\mu = \Lambda^\mu{}_\kappa A^\kappa \Lambda_\mu{}^\nu A_\nu = \delta^\nu_\kappa A^\kappa A_\nu = A^\kappa A_\kappa$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = A^\mu A_\mu$