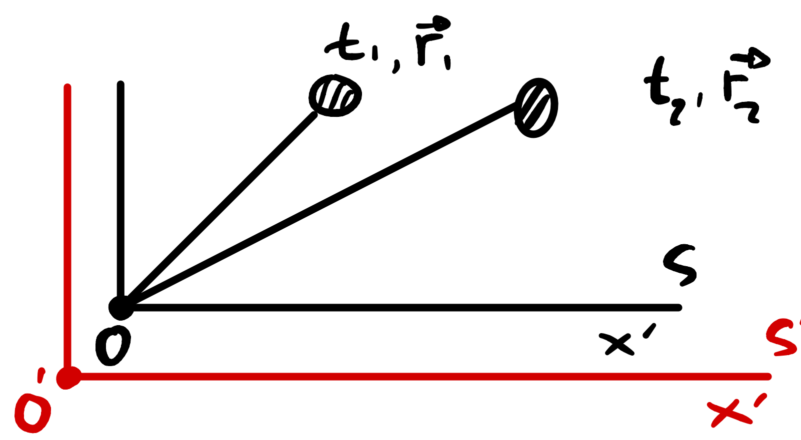
Εισαγωγη

διαλεξη 1η 23.11.2021 (Κ. Θεοφιλατος)

Συστηματα αναφορας



π.χ.

κλασσικο δυναμικο αλληλεπιδρασης

$$U(r_1, r_2) = -\frac{G m_1 m_2}{|\bar{r}_2 - \bar{r}_1|}$$

$$\vec{F}_{21} = -\frac{G m_1 m_2}{|\bar{r}_2 - \bar{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$

(υποθετει απειρη ταχυτητα αλληλεπιδρασης)

Αδρανειακα συστηματα αναφορας

Οταν $\vec{F}_{ext} = 0$ τοτε $\frac{d\vec{r}}{dt} = \sigma\tau\alpha\theta$

Υπαρχουν απειρα ΑΣΑ: οι εξισωσεις κινησης δεν πρεπει να εξαρτωνται απο την επιλογη του ΑΣΑ

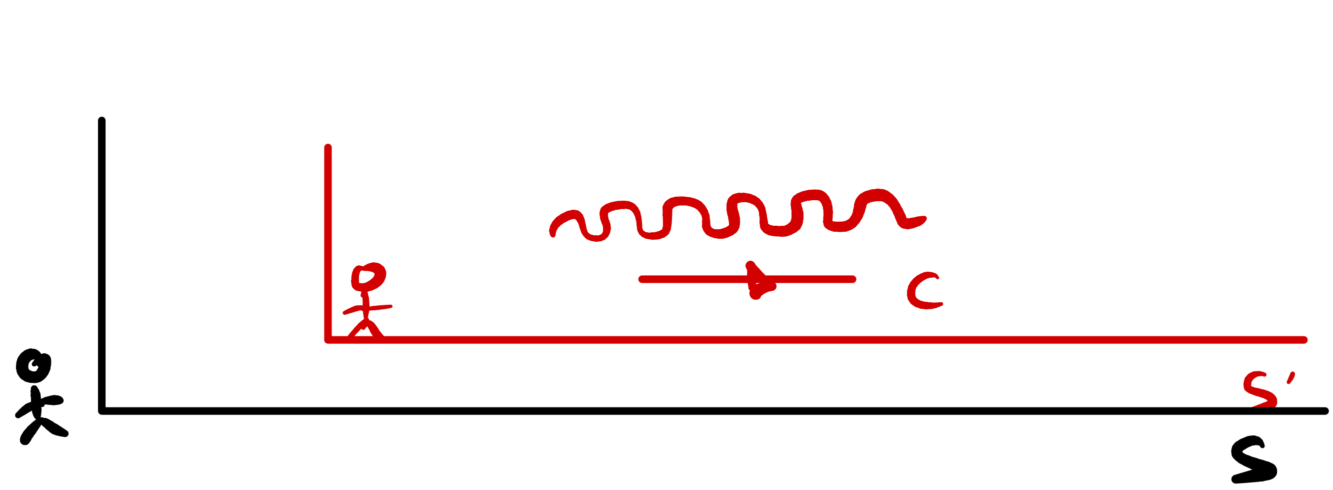
Αξιωμα της σχετικοτητας → ισοδυναμια ΑΣΑ

Πειραματικα γνωριζουμε οτι υπαρχει ενα ανω οριο, μεγιστη ταχυτητα αλληλεπιδρασης, $c = 3 \cdot 10^8\, m/s$

- ιδια σε ολα τα ΑΣΑ (αξιωμα σχετικοτητας)
- κανενα σωμα δε μπορει να ταξιδεψει πιο γρηγορα απο αυτη

$\{\gamma, g, G(?)\}$

$m = 0$
$q = 0$



Τανυστές

$$dx^{\mu'} = \frac{\partial x^{\mu'}}{\partial x^{\mu}} dx^{\mu} \overset{(L.T.)}{=} \Lambda^{\mu'}{}_{\mu}\, dx^{\mu}$$

αντιμεταθετικός (?) — αναλλοίωτος τανυστής 1ης τάξεως contravariant

↑ απειροστό διάνυσμα μετατόπισης

$$\left(A^{\mu'} = \frac{\partial x^{\mu'}}{\partial x^{\mu}} A^{\mu}\right)$$

$A^{\mu} = (A^0, A^1, A^2, A^3)$

$A_{\mu} = (-A^0, A^1, A^2, A^3)$

$A_{\mu} = (A_0, A_1, A_2, A_3)$

$$\begin{pmatrix} A_0 = -A^0 \\ A_i = A^i \end{pmatrix}$$

συναλλοίωτος τανυστής 1ης τάξεως

$$\partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial}{\partial x^{\mu}}$$

$$dx_{\mu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} dx_{\mu} = \Lambda_{\mu'}{}^{\mu} dx_{\mu}$$

$$\begin{pmatrix} dx^{0'} \\ dx^{1'} \\ dx^{2'} \\ dx^{3'} \end{pmatrix} = \begin{bmatrix} \frac{\partial x^{0'}}{\partial x^0} & \frac{\partial x^{0'}}{\partial x^1} & \frac{\partial x^{0'}}{\partial x^2} & \frac{\partial x^{0'}}{\partial x^3} \\ \frac{\partial x^{1'}}{\partial x^0} & \frac{\partial x^{1'}}{\partial x^1} & \frac{\partial x^{1'}}{\partial x^2} & \frac{\partial x^{1'}}{\partial x^3} \\ \frac{\partial x^{2'}}{\partial x^0} & \frac{\partial x^{2'}}{\partial x^1} & \frac{\partial x^{2'}}{\partial x^2} & \frac{\partial x^{2'}}{\partial x^3} \\ \frac{\partial x^{3'}}{\partial x^0} & \frac{\partial x^{3'}}{\partial x^1} & \frac{\partial x^{3'}}{\partial x^2} & \frac{\partial x^{3'}}{\partial x^3} \end{bmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}$$

Jacobian

π.χ. για προώθηση $\vec{v} = v\hat{x}$ $\quad \Lambda(\vec{v} = v\hat{x})$

$t' \equiv x^{0'} = \gamma(x^0 - v x^1)$

$x' \equiv x^{1'} = \gamma(x^1 - v x^0)$

$y' \equiv x^{2'} = x^2$

$z' \equiv x^{3'} = x^3$

$$\frac{\partial x^{0'}}{\partial x^0} = \gamma$$

$$\frac{\partial x^{0'}}{\partial x^1} = -\gamma v$$

$$\begin{pmatrix} dx^{0'} \\ dx^{1'} \\ dx^{2'} \\ dx^{3'} \end{pmatrix} = \begin{bmatrix} \gamma & -\gamma u & 0 & 0 \\ -\gamma u & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} dx^{0} \\ dx^{1} \\ dx^{2} \\ dx^{3} \end{pmatrix}$$

$$dx^{\mu'} = \Lambda^{\mu'}{}_{\mu}\, dx^{\mu}$$

*αναστροφος

T

$$\begin{pmatrix} dx_{0'} \\ dx_{1'} \\ dx_{2'} \\ dx_{3'} \end{pmatrix} = \begin{bmatrix} \frac{\partial x^0}{\partial x^{0'}} & \frac{\partial x^0}{\partial x^{1'}} & \frac{\partial x^0}{\partial x^{2'}} & \frac{\partial x^0}{\partial x^{3'}} \\ \frac{\partial x^1}{\partial x^{0'}} & \frac{\partial x^1}{\partial x^{1'}} & \frac{\partial x^1}{\partial x^{2'}} & \frac{\partial x^1}{\partial x^{3'}} \\ \frac{\partial x^2}{\partial x^{0'}} & \frac{\partial x^2}{\partial x^{1'}} & \frac{\partial x^2}{\partial x^{2'}} & \frac{\partial x^2}{\partial x^{3'}} \\ \frac{\partial x^3}{\partial x^{0'}} & \frac{\partial x^3}{\partial x^{1'}} & \frac{\partial x^3}{\partial x^{2'}} & \frac{\partial x^3}{\partial x^{3'}} \end{bmatrix} \begin{pmatrix} dx_{0} \\ dx_{1} \\ dx_{2} \\ dx_{3} \end{pmatrix}$$

$$dx_{\mu'} = \Lambda_{\mu'}{}^{\mu}\, dx_{\mu}$$

$$\begin{aligned} x^0 &= \gamma(x^{0'} + u x^{1'}) \\ x^1 &= \gamma(x^{1'} + u x^{0'}) \\ x^2 &= x^{2'} \\ x^3 &= x^{3'} \end{aligned}$$

$$\frac{\partial x^0}{\partial x^{0'}} = \gamma$$

$$\frac{\partial x^0}{\partial x^{1'}} = \gamma u$$

$$\begin{pmatrix} dx_{0'} \\ dx_{1'} \\ dx_{2'} \\ dx_{3'} \end{pmatrix} = \begin{bmatrix} \gamma & \gamma u & 0 & 0 \\ \gamma u & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} dx_{0'} \\ dx_{1'} \\ dx_{2'} \\ dx_{3'} \end{pmatrix}$$

$$dx_{\mu'} = \Lambda_{\mu'}{}^{\mu}\, dx_{\mu}$$

$A^{\mu} = (A^0, \vec{A}) \equiv (A^0, A^1, A^2, A^3)$ αντιμεταθετικός τανυστής 1ης τάξεως αν μετασχηματίζεται όπως dx^{μ}

$A_{\mu} = (-A^0, \vec{A})$ συναλλοίωτος τανυστής 1ης τάξεως αν μετασχηματίζεται όπως

$$\partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}}$$

$\vec{A}$ = ευκλείδειο διάνυσμα (μετ. κατάλληλα σε 3-D στροφές και μετατοπίσεις)

$$A^{\mu} \cdot A_{\mu} = A_{\mu'} A^{\mu'} = -(A^0)^2 + (\vec{A} \cdot \vec{A})$$

$$A^{\mu'} \cdot A_{\mu'} = (\Lambda^{\mu'}{}_{\mu} A^{\mu})(\Lambda_{\mu'}{}^{\kappa} A_{\kappa}) = (\Lambda^{\mu'}{}_{\mu} \Lambda_{\mu'}{}^{\kappa}) A^{\mu} A_{\kappa}$$

$$\delta^{\kappa}_{\mu} = \begin{cases} 1 & \mu = \kappa \\ 0 & \mu \neq \kappa \end{cases}$$

$$A^{\mu'} A_{\mu'} = \dots = \delta^{\kappa}_{\mu} A^{\mu} A_{\kappa} = A^{\mu} A_{\mu}$$

δηλ. $\Lambda_{\mu'}{}^{\kappa} = (\Lambda^{-1})^{\kappa}{}_{\mu'}$

Παραδειγμα

Διαλεξη $2^η$ 24.11.2021

Εστω $A^\mu = (A^0, A^1, 0, 0)$ να βρεθουν.

$$\begin{pmatrix} A_0 = -A^0 \\ A_1 = A^1 \end{pmatrix}$$

α) $A_\mu = \begin{bmatrix} -A^0 \\ A^1 \end{bmatrix}$

β) $A^{\mu'} = \begin{bmatrix} \gamma(A^0 - \upsilon A^1) \\ \gamma(A^1 - \upsilon A^0) \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\upsilon \\ -\gamma\upsilon & \gamma \end{bmatrix} \begin{bmatrix} A^0 \\ A^1 \end{bmatrix}$

γ) $A_{\mu'} = \begin{bmatrix} \gamma & \gamma\upsilon \\ \gamma\upsilon & \gamma \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \end{bmatrix}$

για $\Lambda = \Lambda(\vec{\upsilon} = \upsilon\hat{x})$ (τυποποιημενος μετασχ.)

$$A^\mu = \begin{pmatrix} A^0 \\ A^1 \end{pmatrix} \qquad \underbrace{n_{\nu\mu} A^\mu} = A_\nu$$

$$n_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = n^{\mu\nu} \qquad \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A^0 \\ A^1 \end{bmatrix} = \begin{bmatrix} -A^0 \\ A^1 \end{bmatrix}$$

$$\overset{-1}{n_{00}} A^0 + \overset{0}{\cancel{n_{01}}} A^1 + \overset{0}{\cancel{n_{02}}} A^2 + \overset{0}{\cancel{n_{03}}} A^3 = A_0$$

$$\overset{0}{\cancel{n_{10}}} A^0 + \overset{1}{n_{11}} A^1 + \overset{0}{n_{12}} A^2 + \overset{0}{n_{13}} A^3 = A_1$$

$$\dots \qquad = A_2$$

$$= A_4$$

$$A^{\mu'} = \Lambda^{\mu'}{}_{\mu} A^\mu$$

$$\Lambda^{\mu'}{}_{\mu} = \begin{bmatrix} \gamma & -\gamma\upsilon \\ -\gamma\upsilon & \gamma \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad n^{\mu\nu}$$

$$\begin{bmatrix} A^{0'} \\ A^{1'} \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\upsilon \\ -\gamma\upsilon & \gamma \end{bmatrix} \begin{bmatrix} A^0 \\ A^1 \end{bmatrix} = \begin{bmatrix} \gamma(A^0 - \upsilon A^1) \\ \gamma(A^1 - \upsilon A^0) \end{bmatrix}$$

$$A_{\mu'} = n_{\mu'\kappa'} A^{\kappa'}$$

$$A_{0'} = \overset{-1}{n_{0'0'}} A^{0'} + \overset{0}{n_{0'1'}} A^{1'} = -A^{0'} = -\gamma(A^0 - \upsilon A^1)$$

$$A_{1'} = \overset{0}{n_{1'0'}} A^{0'} + \overset{1}{n_{1'1'}} A^{1'} = A^{1'} = \gamma(A^1 - \upsilon A^0)$$

$$\begin{bmatrix} A_{0'} \\ A_{1'} \end{bmatrix} = \begin{bmatrix} -\gamma(-A_0 - \upsilon A_1) \\ \gamma(A_1 + \upsilon A_0) \end{bmatrix} = \begin{bmatrix} \gamma(A_0 + \upsilon A_1) \\ \gamma(A_1 + \upsilon A_0) \end{bmatrix} = \begin{bmatrix} \gamma & \gamma\upsilon \\ \gamma\upsilon & \gamma \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \end{bmatrix}$$

Παραδειγμα

Δειξτε $\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial t^2}\right]\cdot\phi(\vec{x},t)=0$

$$\square=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial t^2}$$

Εστω $\Lambda=\Lambda(\vec{u}=u\hat{x})$ $\quad\square'=?$

$$
\begin{aligned}
t' &= \gamma(t-ux) &&= t'(t,x,y,z)\\
x' &= \gamma(x-ut) &&= x'(t,x,y,z)\\
y' &= y &&= y'(t,x,y,z)\\
z' &= z &&= z'(t,x,y,z)
\end{aligned}
$$

$$
\begin{aligned}
x^0 = t &= \gamma(t'+ux') &&= t(t',x',y',z')\\
x^1 = x &= \gamma(x'+ut') &&= x(t',x',y',z')\\
x^2 = y &= y' &&= y(t',x',y',z')\\
x^3 = z &= z' &&= z(t',x',y',z')
\end{aligned}
$$

$$\frac{\partial t}{\partial t'}=\frac{\partial}{\partial t'}\left[\gamma(t'+ux')\right]=\gamma$$

$$\frac{\partial t}{\partial x'}=\gamma u$$

$$\frac{\partial t}{\partial y'}=\frac{\partial t}{\partial z'}=0$$

$$\frac{\partial x}{\partial t'}=\gamma u \qquad \frac{\partial x}{\partial x'}=\gamma$$

$$\frac{\partial}{\partial t'}=\frac{\partial}{\partial t}\frac{\partial t}{\partial t'}+\frac{\partial}{\partial x}\frac{\partial x}{\partial t'}+\frac{\partial}{\partial y}\frac{\partial y}{\partial t'}+\frac{\partial}{\partial z}\frac{\partial z}{\partial t'}$$

$$\frac{\partial}{\partial x'}=\frac{\partial}{\partial t}\frac{\partial t}{\partial x'}+\frac{\partial}{\partial x}\frac{\partial x}{\partial x'}+\frac{\partial}{\partial y}\frac{\partial y}{\partial x'}+\frac{\partial}{\partial z}\frac{\partial z}{\partial x'}$$

$$\frac{\partial}{\partial y'}=\frac{\partial}{\partial t}\frac{\partial t}{\partial y'}+\frac{\partial}{\partial x}\frac{\partial x}{\partial y'}+\frac{\partial}{\partial y}\frac{\partial y}{\partial y'}+\frac{\partial}{\partial z}\frac{\partial z}{\partial y'}$$

$$\frac{\partial}{\partial z'}=\frac{\partial}{\partial t}\frac{\partial t}{\partial z'}+\frac{\partial}{\partial x}\frac{\partial x}{\partial z'}+\frac{\partial}{\partial y}\frac{\partial y}{\partial z'}+\frac{\partial}{\partial z}\frac{\partial z}{\partial z'}$$

$$
\begin{bmatrix}\frac{\partial}{\partial t'}\\ \frac{\partial}{\partial x'}\\ \frac{\partial}{\partial y'}\\ \frac{\partial}{\partial z'}\end{bmatrix}
=
\begin{bmatrix}
\frac{\partial t}{\partial t'} & \frac{\partial x}{\partial t'} & \frac{\partial y}{\partial t'} & \frac{\partial z}{\partial t'}\\
\frac{\partial t}{\partial x'} & \frac{\partial x}{\partial x'} & \frac{\partial y}{\partial x'} & \frac{\partial z}{\partial x'}\\
\frac{\partial t}{\partial y'} & \frac{\partial x}{\partial y'} & \frac{\partial y}{\partial y'} & \frac{\partial z}{\partial y'}\\
\frac{\partial t}{\partial z'} & \frac{\partial x}{\partial z'} & \frac{\partial y}{\partial z'} & \frac{\partial z}{\partial z'}
\end{bmatrix}
\begin{bmatrix}\frac{\partial}{\partial t}\\ \frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{bmatrix}
$$

$$\begin{bmatrix} \frac{\partial}{\partial t'} \\ \frac{\partial}{\partial x'} \\ \frac{\partial}{\partial y'} \\ \frac{\partial}{\partial z'} \end{bmatrix} = \begin{bmatrix} \gamma & \gamma v & 0 & 0 \\ \gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix}$$

$$\frac{\partial}{\partial y'} = \frac{\partial}{\partial y} \qquad \frac{\partial}{\partial z'} = \frac{\partial}{\partial z}$$

$$\frac{\partial^2}{\partial y'^2} = \frac{\partial^2}{\partial y^2} \qquad \frac{\partial^2}{\partial z'^2} = \frac{\partial^2}{\partial z^2}$$

$$\frac{\partial}{\partial t'} \equiv \partial_{t'} = \gamma \partial_t + \gamma v \partial_x$$

$$\frac{\partial}{\partial x'} \equiv \partial_{x'} = \gamma \partial_x + \gamma v \partial_t$$

$$\partial_{t'}(\partial_{t'}) = (\gamma \partial_t + \gamma v \partial_x)(\gamma \partial_t + \gamma v \partial_x) =$$

$$\frac{\partial^2}{\partial t'^2} = \gamma^2 \partial_t^2 + 2\gamma v \gamma \partial_x \partial_t + \gamma^2 v^2 \partial_x^2$$

$$\frac{\partial^2}{\partial x'^2} = (\gamma \partial x + \gamma v \partial_t)(\gamma \partial x + \gamma v \partial_t)$$

$$\partial_{x'}^2 = (\gamma^2 \partial_x^2 + 2\gamma v \gamma \partial_x \partial_t + \gamma^2 v^2 \partial_t^2)$$

$$\partial_{x'}^2 - \partial_{t'}^2 =$$

$$= \gamma^2 \partial_x^2 + \cancel{2\gamma v \gamma \partial_x \partial_t} + \gamma^2 v^2 \partial_t^2 - \gamma^2 \partial_t^2 - \cancel{2\gamma v \gamma \partial_x \partial_t} - \gamma^2 v^2 \partial_x^2$$

$$= \gamma^2 \partial_x^2 - \gamma^2 v^2 \partial_x^2 - (\gamma^2 \partial_t^2 - \gamma^2 v^2 \partial_t^2)$$

$$= \gamma^2 \overset{1}{(1 - v^2)} \partial_x^2 - \gamma^2 \overset{1}{(1 - v^2)} \partial_t^2 = \partial_x^2 - \partial_t^2$$

$$\gamma^2 = \frac{1}{1 - v^2}$$

$$\Box' = \Box \qquad \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$\Box' \phi' = 0$$

$$\Box \phi = 0$$

$$\partial_\mu = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

$$\partial^\mu = \left(-\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

$$\partial_\mu \partial^\mu \phi = \left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \phi = 0$$

$$\boxed{\partial_{\mu'} \partial^{\mu'} = \partial_\mu \partial^\mu}$$

Τετραδιανύσματα

Διάλεξη 3η (30.11.2021)

Τανυστές $1^{ης}$ τάξεως στις 4D

$x^\mu = (t, \vec{r})$

dx^μ $\boxed{\begin{matrix}\vec{r}, & \frac{d\vec{r}}{dt}, & \frac{d^2\vec{r}}{dt^2} \\ \vec{r}, & \vec{v}, & \vec{a}\end{matrix}}$

$u^\mu \stackrel{?}{=} (\quad)$

$a^\mu \stackrel{?}{=} (\quad)$

διάνυσμα θέσεως → αναλλοίωτο μόνο κάτω από ομογενείς μετ. Lorentz, δηλαδή αυτούς που αφήνουν το (0,0,0,0) στη θέση του

Δοκιμάζω το $u^\mu \stackrel{?}{=} \frac{dx^\mu}{dt} = \frac{d}{dt}(t, x, y, z) = \left(\frac{dt}{dt}, \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) = (1, u_x, u_y, u_z)$

π.χ. $\Lambda(\vec{v} = v\hat{x})$ $\qquad u^{\mu'} \stackrel{?}{=} \left(\frac{dt'}{dt'}, \frac{dx'}{dt'}, \frac{dy'}{dt'}, \frac{dz'}{dt'}\right)$

$dt' = \gamma(dt - v\,dx)$

$dx' = \gamma(dx - v\,dt)$

$dy' = dy$

$dz' = dz$

$$\frac{dx'}{dt'} = \frac{\gamma(dx - v\,dt)}{\gamma(dt - v\,dx)} = \frac{\frac{dx}{dt} - v}{1 - v\cdot\frac{dx}{dt}}$$

$$u_x' = \frac{u_x - v}{1 - v\cdot u_x}$$

$$u_y' = \frac{dy'}{dt'} = \frac{dy}{\gamma(dt - v\,dx)} = \frac{u_y/\gamma}{1 - v\cdot u_x}$$

$$u_z' = \frac{dz'}{dt'} = \frac{dz}{\gamma(dt - v\,dx)} = \frac{u_z/\gamma}{1 - v\cdot u_x}$$

$$u^{\mu'} = \left(1, \frac{u_x - v}{1 - v\cdot u_x}, \frac{u_y/\gamma}{1 - v\cdot u_x}, \frac{u_z/\gamma}{1 - v\cdot u_x}\right)$$

δεν μετ. σαν τανυστής $1^{ης}$ τάξεως

$u^{0'} = \gamma(u^0 - v u^1)$

$u^{1'} = \gamma(u^1 - v u^0)$

$$dt'^2 - (dx')^2 - (dy')^2 - (dz')^2 = (dt)^2 - (dx)^2 - (dy)^2 - (dz)^2$$

$$= (dt)^2\left(1 - \frac{dx^2}{dt^2} - \frac{dy^2}{dt^2} - \frac{dz^2}{dt^2}\right)$$

$$= (dt)^2\left(1 - \vec{v}\cdot\vec{v}\right) = dt^2/\gamma^2$$

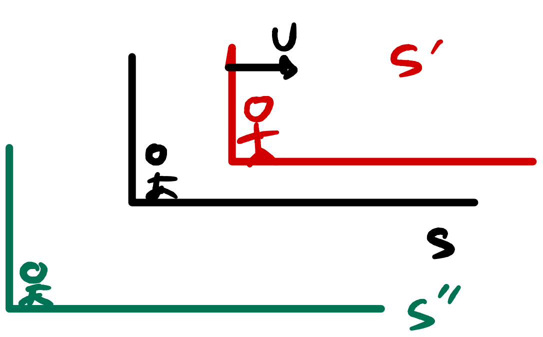
$dt' = d\tau$

$dx' = 0$

$dy' = 0$

$dz' = 0$

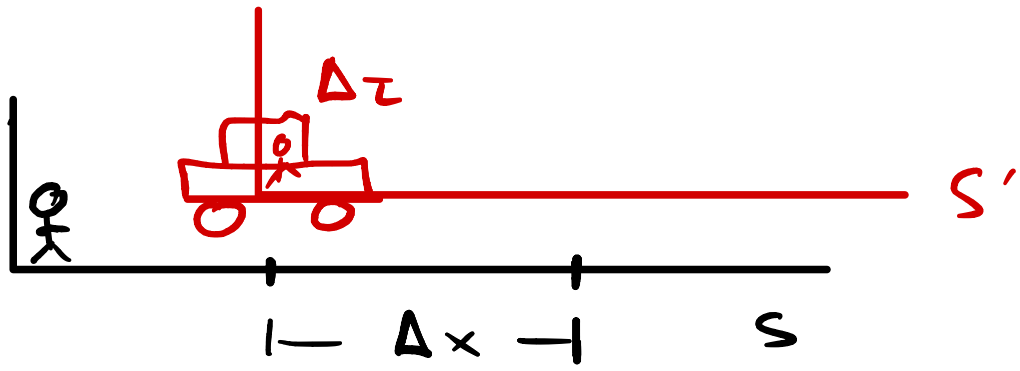
$$\boxed{\gamma\, d\tau = dt}$$



$$U^\mu \equiv \frac{dx^\mu}{d\tau} = \gamma \frac{dx^\mu}{dt} = (\gamma, \gamma \vec{v}) \qquad \frac{\Delta x^\mu}{\Delta \tau}$$

$$U^{\mu'} = \Lambda^{\mu'}{}_{\mu} U^\mu$$

$$U_\mu = (-\gamma, \gamma \vec{v})$$

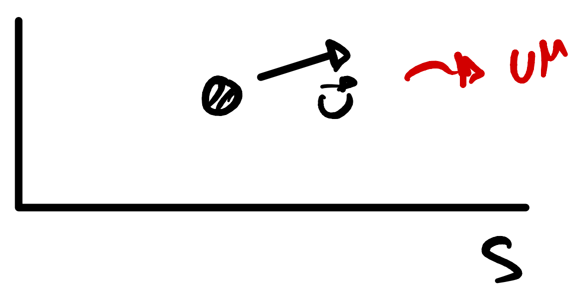


$$U^\mu U_\mu \overset{?}{=} -\gamma^2 + \gamma^2 \vec{v}\cdot\vec{v} = -\gamma^2 + \gamma^2 v^2 = -\gamma^2(1 - v^2) = \frac{-1}{1-v^2}(1-v^2) = -1$$

$|\vec{v}| = v$

$$\boxed{U^\mu \cdot U_\mu = U_{\mu'} \cdot U^{\mu'} = -1}$$

$$U^\mu = (\gamma, \gamma \vec{v})$$



$$U^{\mu'} = (\gamma', \gamma' \vec{v}') = (1, \vec{0})$$

S' σύστημα ηρεμίας

$$\boxed{U^\mu \cdot U_\mu = U^{\mu'} \cdot U_{\mu'} = (-1, \vec{0}) \cdot (1, \vec{0}) = -1}$$

$$a^\mu = \frac{dU^\mu}{d\tau} = \gamma \frac{dU^\mu}{dt} = \gamma \frac{d}{dt}(\gamma, \gamma \vec{v})$$

$$\dot{\gamma} = \frac{d}{dt}(\gamma) = \frac{d}{dt}\left([1 - \vec{v}\cdot\vec{v}]^{-1/2}\right) = -\frac{1}{2}(1 - \vec{v}\cdot\vec{v})^{-3/2}\left(-2\vec{v}\cdot\frac{d\vec{v}}{dt}\right)$$

$$\boxed{\dot{\gamma} = \gamma^3(\vec{v}\cdot\vec{a})}$$

$$a^\mu = \gamma \frac{d}{dt}(\gamma, \gamma \vec{v}) = \gamma\left(\gamma^3(\vec{v}\cdot\vec{a}), \frac{d}{dt}(\gamma \vec{v})\right) =$$

$$\frac{d}{dt}(\gamma \vec{v}) = \dot{\gamma}\vec{v} + \gamma \cdot \frac{d\vec{v}}{dt} = \gamma^3(\vec{v}\cdot\vec{a})\vec{v} + \gamma \cdot \vec{a}$$

$$a^\mu = \left(\gamma^4(\vec{v}\cdot\vec{a}), \gamma^4(\vec{v}\cdot\vec{a})\cdot\vec{v} + \gamma^2 \vec{a}\right)$$

$$a^\mu a_\mu \equiv (\vec{a}_0)^2$$

↑ ιδιοεπιτάχυνση

$$a^\mu a_\mu = \left(\gamma^4(\vec{u}\cdot\vec{a}),\ \gamma^4(\vec{u}\cdot\vec{a})\cdot\vec{u} + \gamma^2\vec{a}\right)\left(-\gamma^4(\vec{u}\cdot\vec{a}),\ \gamma^4(\vec{u}\cdot\vec{a})\cdot\vec{u} + \gamma^2\vec{a}\right)$$

$$= -\gamma^8(\vec{u}\cdot\vec{a})^2 + \gamma^8(\vec{u}\cdot\vec{a})^2\cdot u^2 + 2\gamma^6(\vec{u}\cdot\vec{a})^2 + \gamma^4(\vec{a})^2$$

$$= -\gamma^8(\vec{u}\cdot\vec{a})^2\big(\overbrace{1-u^2}^{\gamma^{-2}}\big) + 2\gamma^6(\vec{u}\cdot\vec{a})^2 + \gamma^4(\vec{a})^2$$

$$= -\gamma^6(\vec{u}\cdot\vec{a})^2 + 2\gamma^6(\vec{u}\cdot\vec{a})^2 + \gamma^4(\vec{a})^2$$

$$\boxed{a^\mu a_\mu = \gamma^6(\vec{u}\cdot\vec{a})^2 + \gamma^4(\vec{a})^2} \equiv (\vec{a}_0)^2$$

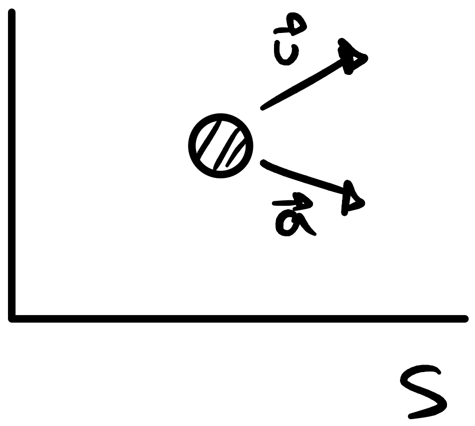
ιδιοεπιτάχυνση

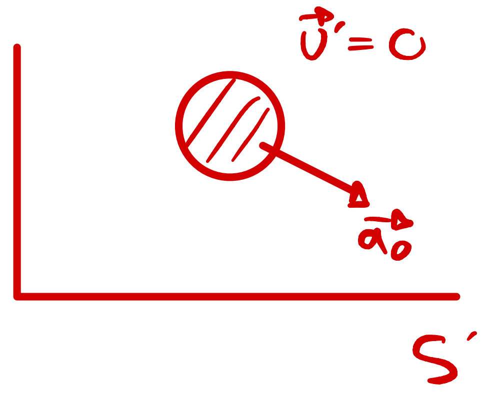
- $\vec{a} \parallel \vec{u}$ τότε

$$a^\mu a_\mu = \gamma^6 u^2 a^2 + \gamma^4 a^2$$
$$= \gamma^4 a^2(1 + \gamma^2 u^2)$$
$$= \gamma^4 a^2\left(\frac{1-u^2}{1-u^2} + \frac{u^2}{1-u^2}\right)$$
$$= \gamma^6 a^2 = \gamma^6(\vec{a})^2$$
$$|\vec{a}_0| = \gamma^3|\vec{a}|$$

- $\vec{a} \perp \vec{u}$ τότε

$$a_\mu a^\mu = \gamma^4(\vec{a})^2 = \gamma^4 a^2$$
$$|\vec{a}_0| = \gamma^2|\vec{a}|$$





στιγμιαίο σύστημα αναφοράς όπου το σώμα είναι σε ηρεμία

$\gamma' = 1$
$\vec{u}' = 0$

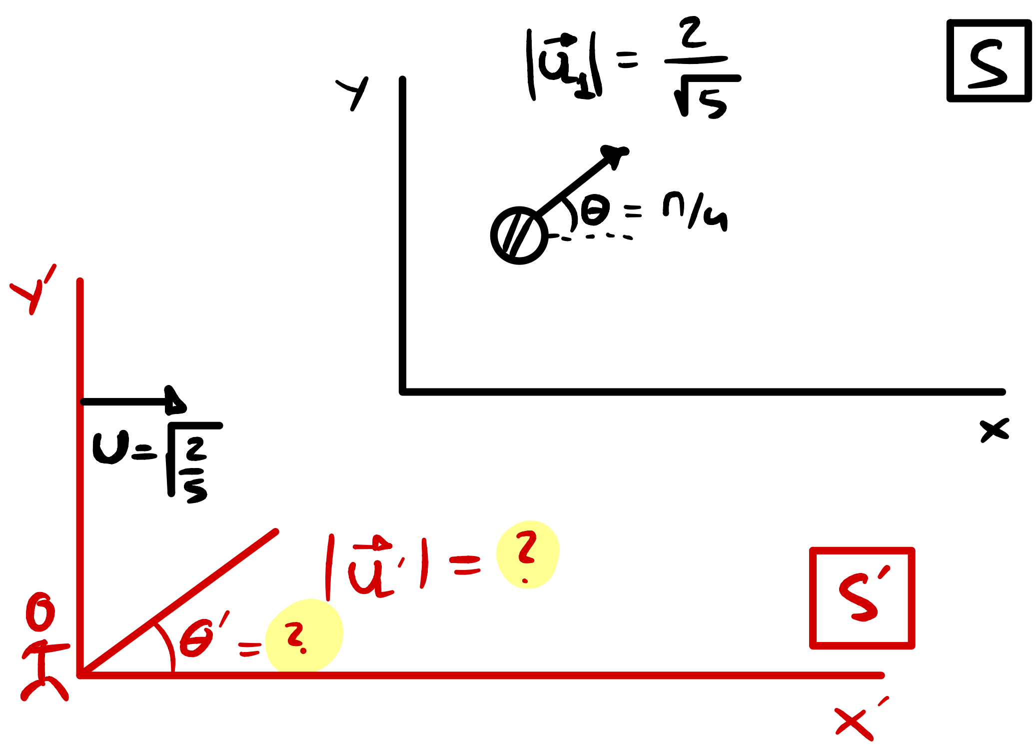
$$a^\mu = \left(\gamma^4(\vec{u}\cdot\vec{a}),\ \gamma^4(\vec{u}\cdot\vec{a})\cdot\vec{u} + \gamma^2\vec{a}\right)$$
$$u^\mu = (\gamma,\ \gamma\vec{u})$$

$$a^{\mu'} = (0,\ \vec{a}_0)$$
$$u^{\mu'} = (1,\ \vec{0})$$

$$\boxed{a^\mu \cdot u_\mu = a^{\mu'} u_{\mu'} = 0}$$

$$\frac{d}{d\tau}(u^\mu \cdot u_\mu) = \frac{d}{d\tau}(-1)$$
$$u^\mu \cdot a_\mu + a^\mu \cdot u_\mu = 0$$
$$2u^\mu \cdot a_\mu = 2u_\mu \cdot a^\mu = 0$$

Προβλημα 10.23 (Griffiths) —— Διαλεξη 4η (01.12.2021)



$\Lambda(\vec{v} = \sqrt{\frac{2}{5}}\,\hat{x})$

$\hat{x}' = \hat{x}$
$\hat{y}' = \hat{y}$
$\hat{z}' = \hat{z}$

α τροπος

$$\vec{u}_1 = \left(\frac{\sqrt{2}}{\sqrt{5}}, \frac{\sqrt{2}}{\sqrt{5}}, 0\right)$$

$$\gamma_1 = \frac{1}{\sqrt{1-4/5}} =$$

$$u_1^\mu = (\sqrt{5}, \sqrt{2}, \sqrt{2}, 0)$$

$$u_1^{\mu'} = \begin{pmatrix} \sqrt{\frac{5}{3}} & -\sqrt{\frac{2}{3}} & 0 & 0 \\ -\sqrt{\frac{2}{3}} & \sqrt{\frac{5}{3}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} \sqrt{5} \\ \sqrt{2} \\ \sqrt{2} \\ 0 \end{bmatrix}$$

$$\gamma = \frac{1}{\sqrt{1-2/5}} = \sqrt{\frac{5}{3}}$$

$$\gamma v = \sqrt{\frac{5}{3}}\cdot\sqrt{\frac{2}{5}} = \sqrt{\frac{2}{3}}$$

$$= \begin{bmatrix} 5/\sqrt{3} - \frac{2}{\sqrt{3}} \\ -\frac{\sqrt{10}}{\sqrt{3}} + \frac{\sqrt{10}}{\sqrt{3}} \\ \sqrt{2} \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{3} \\ 0 \\ \sqrt{2} \\ 0 \end{bmatrix} = \gamma_1' \begin{bmatrix} 1 \\ u_{1x}' \\ u_{1y}' \\ u_{1z}' \end{bmatrix}$$

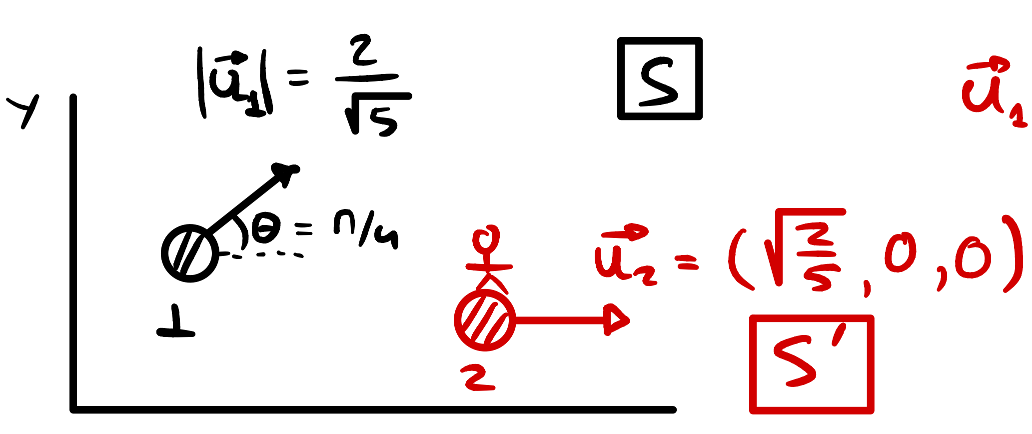
$u_1^{\mu'} \cdot u_{1\mu}' = -1$
$-3 + 2 = -1$
✓ OK

$\gamma_1' = \sqrt{3}$
$\gamma_1' u_{1x}' = 0$
$\gamma_1' u_{1y}' = \sqrt{2}$
$\gamma_1' u_{1z}' = 0$

$u_{1x}' = 0$
$u_{1y}' = \sqrt{2}/\sqrt{3}$
$u_{1z}' = 0$

$\theta' = \frac{\pi}{2}$
$|\vec{u}_1'| = \sqrt{2}/\sqrt{3}$

β τροπος



$\vec{u}_{12} = \vec{u}_1'$

u_1^μ	u_2^μ	$\boxed{S}$
$u_1^{\mu'}$	$u_2^{\mu'}$	$\boxed{S'}$

$$u_1^\mu \cdot u_{2\mu} = u_1^{\mu'} \cdot u_{2\mu'}$$

$$\gamma_2 = \frac{1}{\sqrt{1-2/5}} = \sqrt{5/3}$$

$$u_1^\mu = \left(\sqrt{5}, \sqrt{2}, \sqrt{2}, 0\right) \qquad u_1^{\mu'} = \left(\gamma_1', \gamma_1' \vec{v}_1'\right)$$

$$u_2^\mu = \left(\frac{\sqrt{5}}{\sqrt{3}}, \sqrt{\frac{2}{3}}, 0, 0\right) \qquad u_2^{\mu'} = \left(1, 0\right)$$

$$u_1^\mu \cdot u_{2\mu} = u_1^{\mu'} \cdot u_{2\mu'}$$

$$\left(-\sqrt{5}\sqrt{\frac{5}{3}} + \sqrt{2}\sqrt{\frac{2}{3}} + 0 \cdot \sqrt{2} + 0 \cdot 0\right) = -\gamma_1' + 0$$

$$\gamma_1' = \sqrt{3} = \frac{1}{\sqrt{1 - (\vec{v}_1')^2}} =$$

$$|\vec{u}_1'|^2 = \frac{2}{3}$$

$$u_2^\mu = \begin{bmatrix} \sqrt{5/3} \\ \sqrt{2/3} \\ 0 \\ 0 \end{bmatrix} \qquad u_2^{\mu'} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} \sqrt{\frac{5}{3}} & -\sqrt{\frac{2}{3}} & 0 & 0 \\ -\sqrt{\frac{2}{3}} & \sqrt{\frac{5}{3}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \end{pmatrix} \begin{bmatrix} \sqrt{5/3} \\ \sqrt{2/3} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \checkmark$$

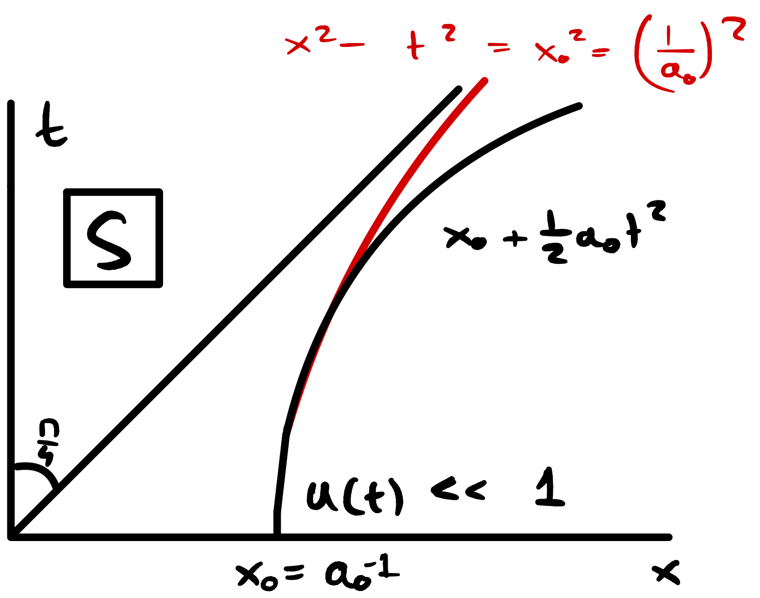
Υπερβολική κίνηση

κίνηση σώματος με σταθερή ιδιοεπιτάχυνση $\vec{a}_0 = a_0 \hat{x}$

Κλασσική Μηχ.

$$\boxed{\begin{aligned} x(t) &= x_0 + \tfrac{1}{2} a t^2 \\ u_x(t) &= a_0 t + \cancel{u_0}^{\,\to 0} \end{aligned}}$$

$$u_y(t) = u_z(t) = 0$$



στιγμιαίο σύστημα αναφοράς

$$a^{\mu'} = (0, a_0, 0, 0)$$

$$a^{\mu} = \left(\gamma^4 (\vec{v}\cdot\vec{a}),\ \gamma^4(\vec{v}\cdot\vec{a})\,\vec{v} + \gamma^2 \vec{a} \right)$$

$u = v$

$$a^{\mu'} a_{\mu'} = a_0^2 \overset{\vec{v}\parallel\vec{a}}{=} (\gamma^3 a)^2$$

$$\vec{a} = (a, 0, 0) \qquad \vec{v} = (v, 0, 0) \qquad \boxed{S}$$

$$a_0 = \gamma^3 a = \frac{d}{dt}(\gamma \vec{v})\Big|_x$$

απόδειξη:

$$\frac{d}{dt}(\gamma \vec{v}) = \dot{\gamma}\vec{v} + \gamma \cdot \frac{d\vec{v}}{dt} = \gamma^3 (\vec{v}\cdot\vec{a})\,\vec{v} + \gamma \cdot \vec{a}$$

$$\underset{x-\text{συνιστώσα}}{\overset{\vec{v}\parallel\vec{a}}{=}} \gamma^3 v^2 a + \gamma a = \gamma(1 + \gamma^2 v^2) a = \gamma\left(\frac{1-v^2}{1-v^2} + \frac{v^2}{1-v^2}\right) a$$

$$= \gamma^3 a$$

$$\frac{d}{dt}(\gamma v) = a_0$$

$$\gamma v = a_0 t + \cancel{v_0}^{\,\to 0}$$

$$(\gamma v)^2 = (a_0 t)^2$$

$$v^2 = (a_0 t)^2 (1 - v^2)$$

$$(1 + (a_0 t)^2)\, v^2 = (a_0 t)^2$$

$$v(t) = \frac{a_0 t}{\sqrt{1 + (a_0 t)^2}}$$

$$\lim_{a_0 t \to 0} v(t) \simeq a_0 t$$

$$\lim_{t \to \infty} v(t) = 1$$

$$\int dx = \int \frac{a_0 t}{\sqrt{1 + (a_0 t)^2}} \, dt = \int \frac{w}{\sqrt{1+w^2}} \frac{dw}{a_0}$$

$$= \int \left(\sqrt{1+w^2}\right)' \frac{dw}{a_0}$$

$w = a_0 t$

$dw = a_0 dt$

$dt = \frac{dw}{a_0}$

$$x(t) = \frac{1}{a_0}\sqrt{1+(a_0 t)^2} + \cancel{\sigma\tau\alpha\vartheta}^{\,0}$$

$$x(0) = x_0 = \frac{1}{a_0}$$

$[\upsilon]$ = αριθμος

$[a_0]$ = μηκος^{-1}

$$x^2 = \left(\frac{1}{a_0}\right)^2 + t^2$$

$$x^2 - t^2 = \left(\frac{1}{a_0}\right)^2$$

εξ. υπερβολης

$$\gamma d\tau = dt$$

$$d\tau = \sqrt{(1-\upsilon^2)}\, dt$$

$$d\tau^2 = \left(1 - \frac{(a_0 t)^2}{1+(a_0 t)^2}\right) dt^2$$

$$\int_0^{\tau} d\tau' = \int_0^{t} \frac{1}{\sqrt{1+(a_0 t')^2}} dt' = \int_0^{\operatorname{arsinh}(a_0 t)} \frac{\cosh w}{\sqrt{1+\sinh^2 w}} \frac{dw}{a_0}$$

$\cosh^2 x - \sinh^2 x = 1$

$\cosh = \frac{1}{2}(e^x + e^{-x})$

$\sinh = \frac{1}{2}(e^x - e^{-x})$

$\sinh w = a_0 t'$

$\cosh w \, dw = a_0 dt'$

$dt = \frac{\cosh w}{a_0} dw$

$$\tau(t) = \frac{1}{a_0} \operatorname{arsinh}(a_0 t)$$

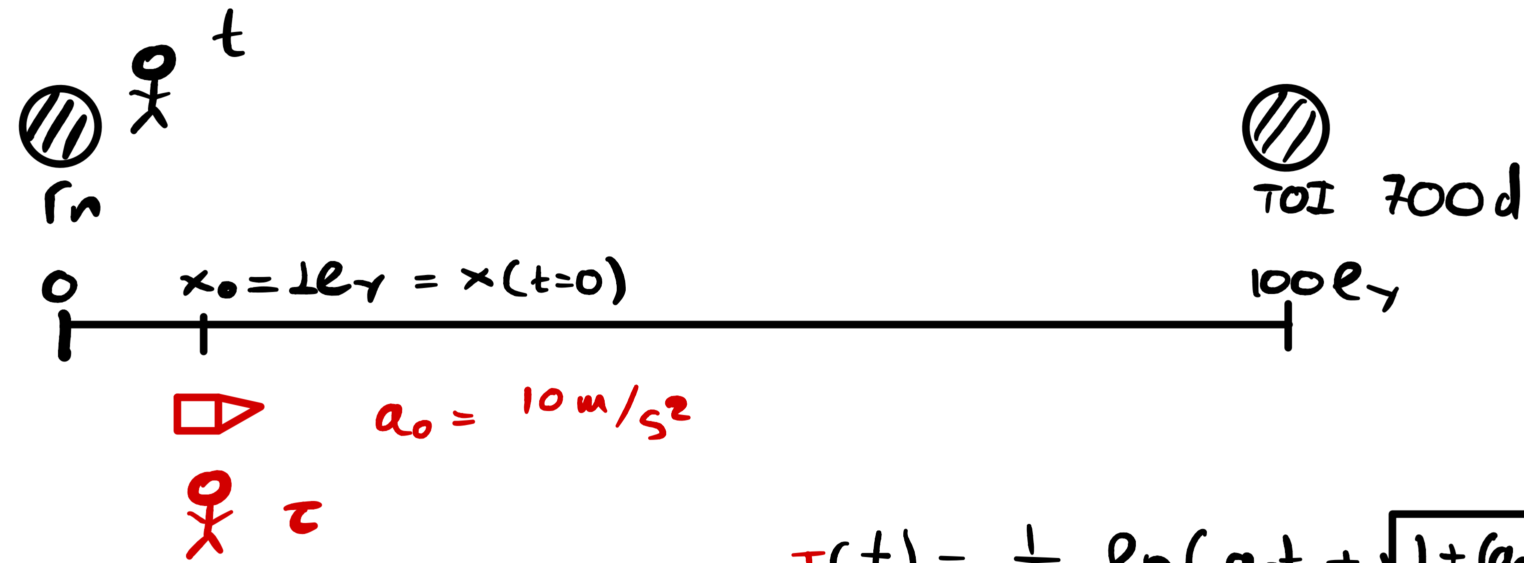
$$\operatorname{arsinh} x = \ln(x + \sqrt{1+x^2})$$

t'	0	t
w	0	$\operatorname{arsinh}(a_0 t)$

$$\tau(t) = \frac{1}{a_0} \ln\left(a_0 t + \sqrt{1+(a_0 t)^2}\right)$$

Ταξίδι στον TOI 700d

5η διάλεξη 07.12.2021



$$\tau(t) = \frac{1}{a_0} \ln\left(a_0 t + \sqrt{1+(a_0 t)^2}\right)$$

t_{100} = ο χρόνος που ο [figure] υπολογίζει ότι ο [figure] έφτασε

$$x^2 - t^2 = \left(\frac{1}{a_0}\right)^2 \xrightarrow{S.I.} x^2 - (ct)^2 = \left(\frac{c^2}{a_0}\right)^2$$

$$t_{100} = \sqrt{\left(-\left(\frac{c^2}{a_0}\right)^2 + d^2\right)} \Big/ c \simeq 99.9 y \quad (3.15 \times 10^9 s)$$

με εκκίνηση από την θέση $x(0) = x_0 = c^2/a_0 = 0.95\ ly$

$d^2 - (ct_{100})^2 = \left(\frac{c^2}{a_0}\right)^2$

$(ct_{100})^2 = d^2 - (c^2/a_0)^2$

$t_{100} = \frac{1}{c}\sqrt{d^2 - (c^2/a_0)^2}$

$$\tau_{100}(t) = \frac{c}{a_0} \ln\left(\frac{a_0 t_{100}}{c} + \sqrt{1 + \left(\frac{a_0 t_{100}}{c}\right)^2}\right) \simeq \ldots 5y$$

$$\varepsilon = \frac{a_0 t}{c} \qquad \tau(\varepsilon) = \frac{c}{a_0} \ln\left(\varepsilon + \sqrt{1+\varepsilon^2}\right)$$

για $\varepsilon \ll 1$

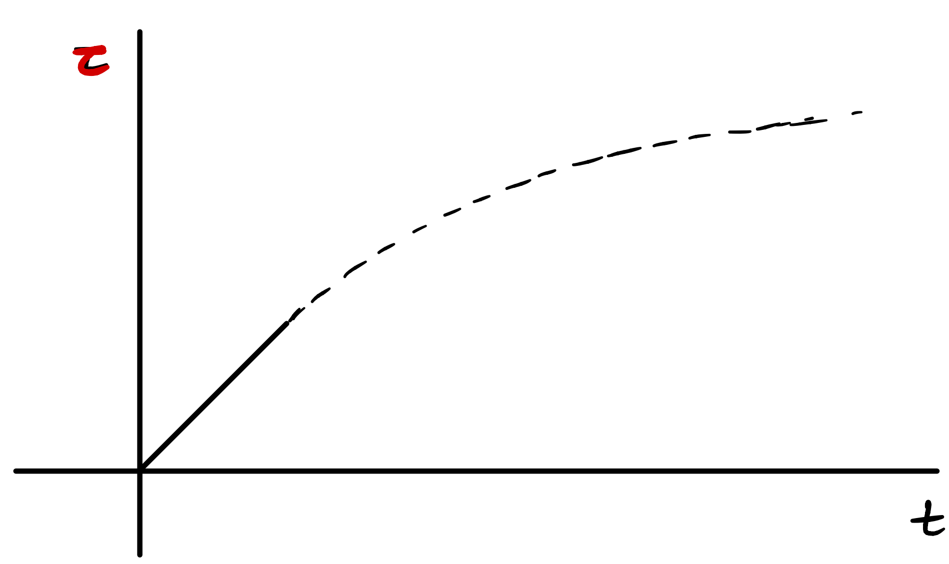
$$\tau'(\varepsilon) = \frac{c}{a_0} \frac{1}{\varepsilon + \sqrt{1+\varepsilon^2}} \left(1 + \frac{\varepsilon}{\sqrt{1+\varepsilon^2}}\right)$$

$$\tau(\varepsilon) \simeq \tau(0) + \varepsilon \cdot \tau'(0) + \frac{1}{2!}\varepsilon^2 \tau''(0) + \ldots$$

$$\simeq 0 + \varepsilon \cdot \frac{c}{a_0} = t$$

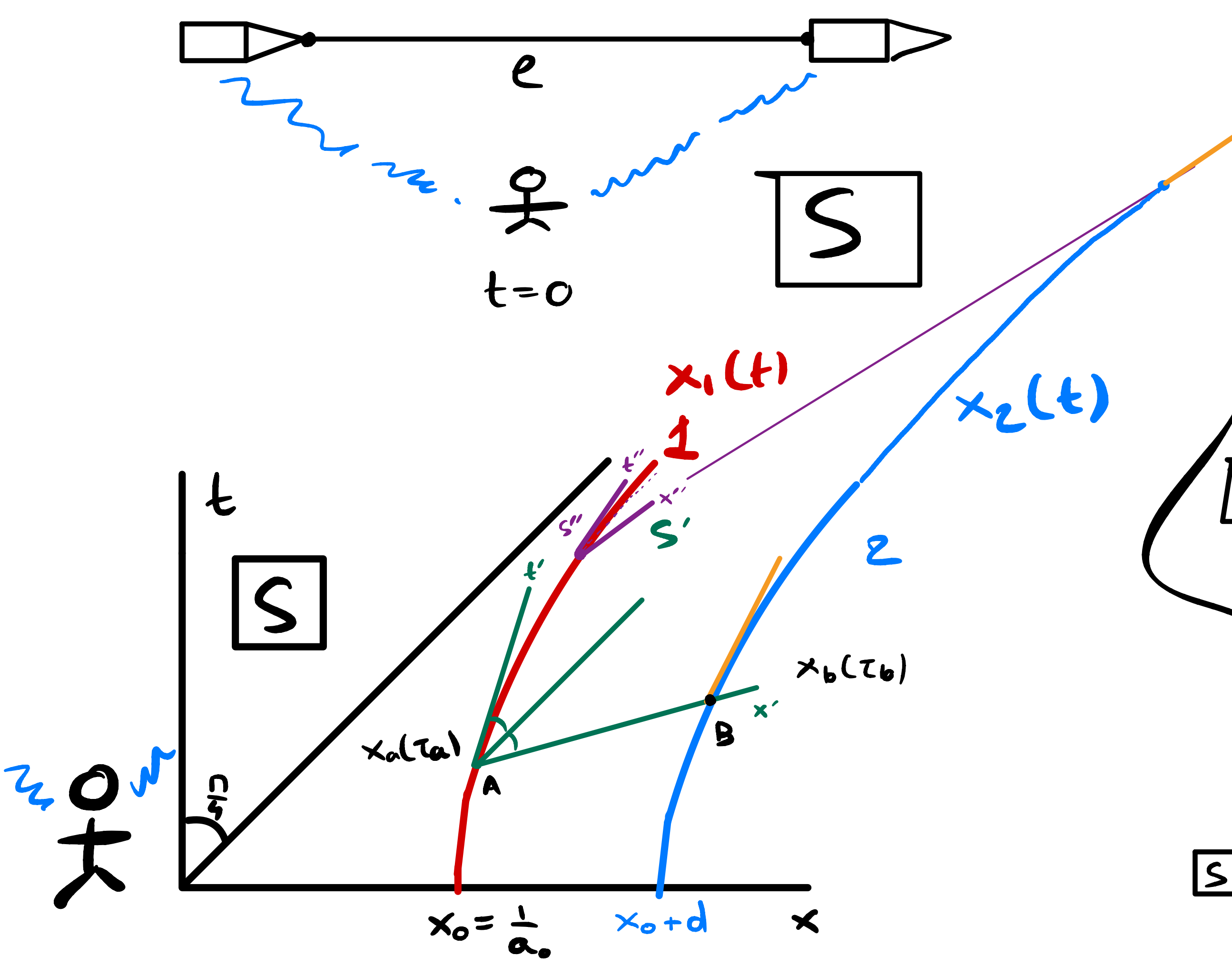
για $\varepsilon \gg 1$

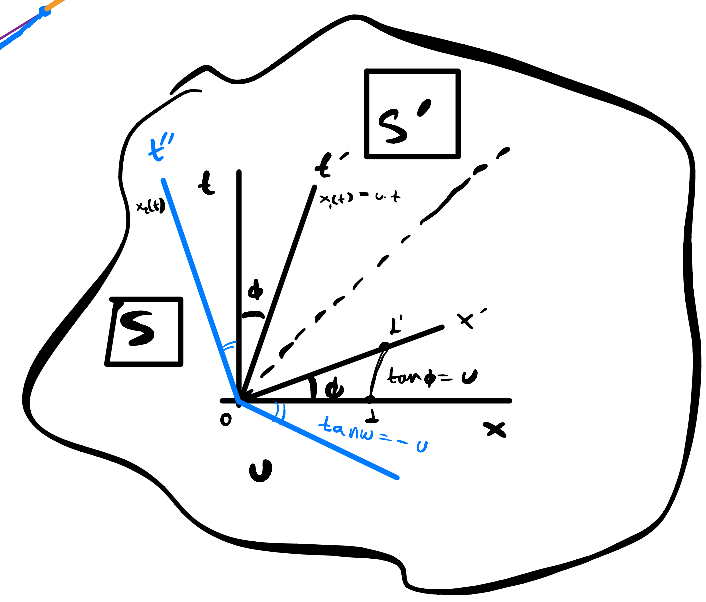
$$\tau(\varepsilon) = \frac{c}{a_0} \ln\left(\varepsilon + \sqrt{1+\varepsilon^2}\right) \simeq \frac{c}{a_0} \ln(2\varepsilon)$$

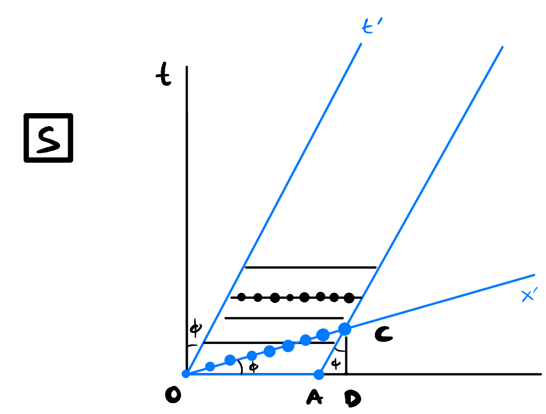


μπορεί το $\varepsilon = \frac{a_0 t}{c}$ να γίνει $\varepsilon \gg 1$?

Παραδοξο του Bell







1 : $x_1^2 - t^2 = \left(\frac{1}{a_0}\right)^2 = x_0^2$

2 : $\cancel{x_2^2 - t^2 = (x_0 + d)^2}$ ⟵ λαθος διοτι $x_2 = x_1 - d$

$$u(t) = \frac{a_0 t}{\sqrt{1 + (a_0 t)^2}} = u_1(t) = u_2(t)$$

1: $x_1(t) = \frac{1}{a_0}\sqrt{1 + (a_0 t)^2}$

2: $x_2(t) = \frac{1}{a_0}\sqrt{1 + (a_0 t)^2} + d$

$x_2(t) - x_1(t) = d$

$\frac{dx_2}{dt} = \frac{dx_1}{dt} = \frac{a_0 t}{\sqrt{1 + (a_0 t)^2}}$

Ορμη

6η διαλεξη 08.12.21

$\vec{P}_{KM} = m\vec{u}$ KM $\longrightarrow$ $p^\mu = m \cdot u^\mu$ (4-ορμη)

$= m(\gamma, \gamma\vec{u})$

$= (m\gamma, m\gamma\vec{u}) = (E, \vec{P})$

$\vec{P} = m\gamma\vec{u}$ (3-ορμη)

$\lim_{u\to 0} \vec{P} \simeq \vec{P}_{KM}$

$E = \gamma m$ (μηδενικη συνιστωσα του p^μ)

$E = \dfrac{m}{\sqrt{1-u^2}} \simeq m\left(1 + \frac{1}{2}\cdot x + \dots\right)$

$f(x) = \dfrac{1}{\sqrt{1-x}}$ | $f(x) = f(0) + x\cdot f'(0) + \dots$

$x = u^2$

$f'(x) = -\frac{1}{2}(1-x)^{-3/2}\cdot(-1) = \frac{1}{2}(1-x)^{-3/2}$

$f(0) = 1 \qquad f'(0) = \frac{1}{2}$

$E = m + \dfrac{mu^2}{2} + \dots$

$= m + \dfrac{P_{KM}^2}{2m} + \dots$

$T \equiv E - m$

$m_e = 511$ keV (keV 2 Joule)

$m_p \simeq 0.938$ GeV $\simeq 1$ GeV

$q\cdot\Delta V = 1.6\cdot 10^{-19}\, C \cdot 1\, Volt$

$= 1\, eV$

$1eV = 1.6\cdot 10^{-19}$ Joule

$\Delta V = 1$

Αν $|\vec{P}| = 6500$ GeV

$u = ?$

Μετρο p^μ ?

$p^\mu \cdot p_\mu \overset{?}{=} -m^2 = (\gamma m, m\gamma\vec{u})\cdot(-\gamma m, m\gamma\vec{u})$

$= -\gamma^2 m^2 + m^2\gamma^2 u^2$

$= -\gamma^2 m^2(1 - u^2)$

$= -m^2 \qquad (-+++)$

$p^\mu = (E, \vec{P})$ $\qquad$ $p_\mu p^\mu = -E^2 + |\vec{P}|^2$

$p_\mu = (-E, \vec{P})$ $\qquad = -E^2 + p^2$

$-E^2 + p^2 = -m^2 \longrightarrow \boxed{E^2 = m^2 + p^2}$ $\qquad \boxed{\vec{u} = \dfrac{\vec{P}}{E}}$

$p^\mu \cdot p_\mu = mu^\mu \cdot mu_\mu = -m^2$ $\qquad [p^2 = m^2 \quad (+---)]$

$\boxed{u^\mu \cdot u_\mu = -1}$

$\vec{P} = \dfrac{m\vec{u}}{\sqrt{1-u^2}} \qquad m = m_0$

$p^\mu =$ φωτ. ?

$E_\gamma = h\cdot\nu \qquad [h] = J\cdot s$

$|\vec{P}_\gamma| = h/\lambda \qquad [\nu] = s^{-1} = Hz$

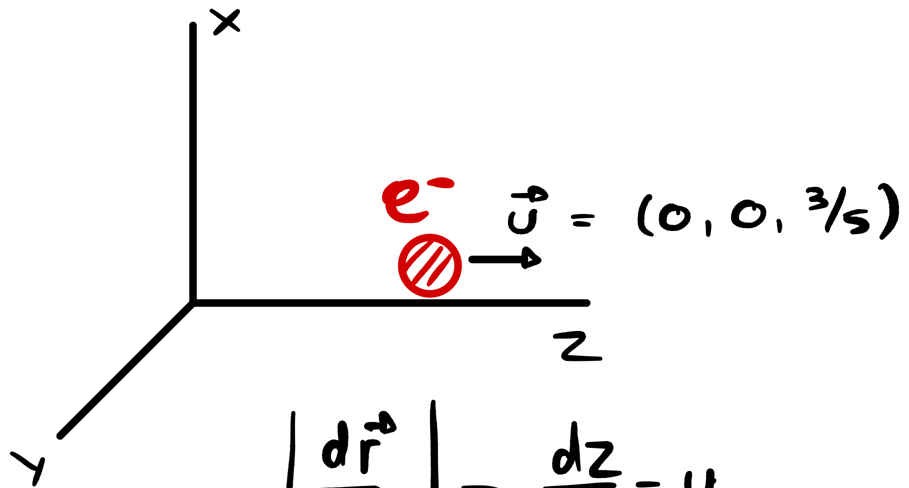
$\lambda\cdot\nu = c = 1$

$\hat{n} = \dfrac{\vec{P}}{|\vec{P}|}$

$$p^\mu \quad \Big| \quad m>0: \quad \begin{bmatrix} m\gamma \\ m\gamma\vec{v} \end{bmatrix} = \begin{bmatrix} E \\ \vec{p} \end{bmatrix} = \begin{bmatrix} \sqrt{m^2+p^2} \\ \vec{p} \end{bmatrix} \quad \Big| \quad m=0: \quad \begin{bmatrix} E_\gamma \\ \vec{p}_\gamma \end{bmatrix} = \begin{bmatrix} h\nu \\ h\nu\,\hat{n} \end{bmatrix} = \begin{bmatrix} \sqrt{\cancel{m^2}+p^2} \\ \vec{p} \end{bmatrix} = \begin{bmatrix} p \\ \vec{p} \end{bmatrix} = \begin{bmatrix} E \\ E\hat{n} \end{bmatrix}$$

$p^\mu(e^-)$

$\mu \in \quad u_z = 3/5 \,, \; m_{e^-}$



$$\gamma = \frac{1}{\sqrt{1-(3/5)^2}} = 5/4$$

$$\gamma u = 5/4 \cdot \frac{3}{5} = 3/4$$

$$m_e \;(=511 \text{ keV})$$

$$\left|\frac{d\vec{r}}{dt}\right| = \frac{dz}{dt} = u_z$$

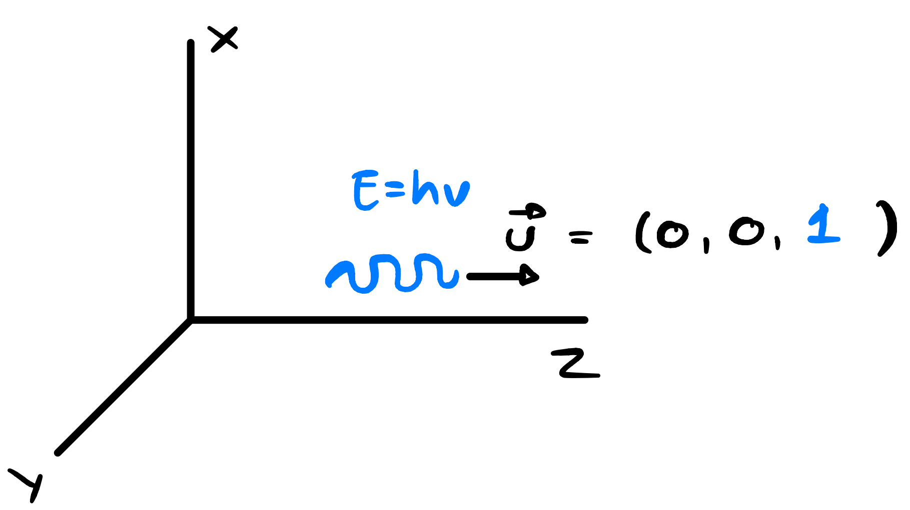
$$p^\mu = \left(\frac{5m}{4}, 0, 0, \frac{3m}{4}\right)$$

$$p^\mu \cdot p_\mu \stackrel{?}{=} -m^2 = -m^2\left(\frac{25}{16} - \frac{9}{16}\right)$$

$$\boxed{\vec{v} = \frac{\vec{p}}{E}} = \frac{\gamma m \vec{v}}{\gamma m}$$

$p^\mu(\gamma)$

$u = 1 \,, \; m = 0 \,, \; E = h\nu \,, \; \hat{n} = (0, 0, 1)$



$$p^\mu(\gamma) = (h\nu, 0, 0, h\nu) = \begin{bmatrix} E \\ 0 \\ 0 \\ E \end{bmatrix}$$

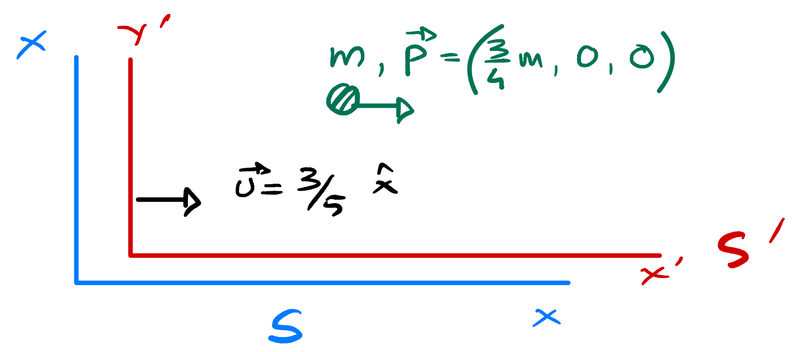
7η διαλεξη 14.12.2021

Ασκηση 1

Αν $p^\mu = \left(\frac{5m}{4}, \frac{3m}{4}, 0, 0\right)$

$p^{\mu'} = \Lambda^{\mu'}{}_{\mu}\left(\vec{v} = \frac{3}{5}\hat{x}\right) p^\mu$ (ΤΜΛ)

$p^{\mu'} = ?$



$p^{\mu'} = \Lambda^{\mu'}{}_{\mu}\, p^\mu \quad | \quad \underset{\boxed{S'}}{P'} = \Lambda \cdot \underset{\boxed{S}}{P}$

$p^\mu = \left(\frac{5m}{4}, \frac{3m}{4}, 0, 0\right)$

$p^{\mu'} = \left(E', p_x', p_y', p_z'\right)$

$E' = ?$

a) $5/4\ m$

b) $2m$

c) m

d) $\frac{3}{5}m$

$\vec{u} = \frac{\vec{p}}{E} = \frac{\gamma_u m \vec{u}}{\gamma_u m} = \left(\frac{3}{5}, 0, 0\right)$

$\gamma_u = \frac{1}{\sqrt{1-\left(\frac{3}{5}\right)^2}} = 5/4$

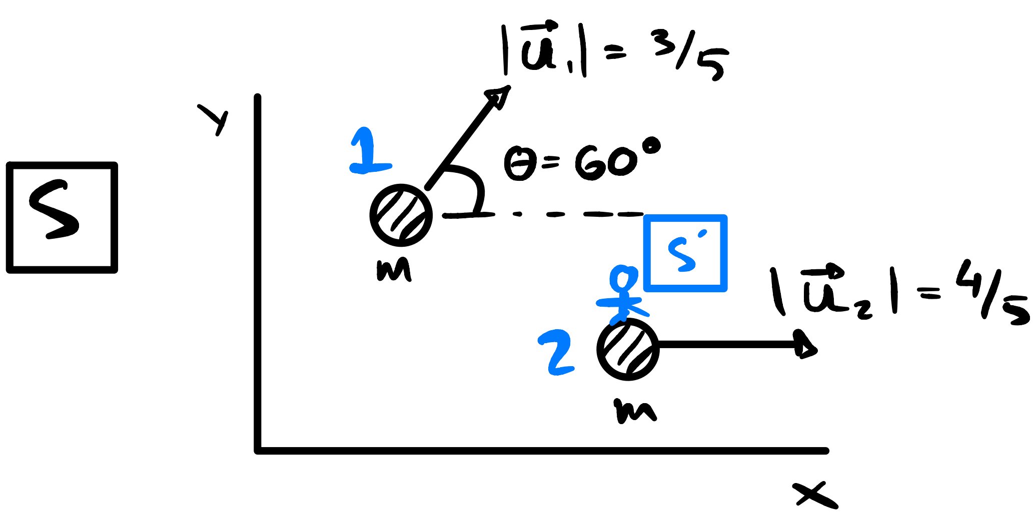
$\boxed{S}: \quad E^2 = p^2 + m^2 \qquad \gamma = \gamma_u = 5/4 \qquad v = u = \frac{3}{5}$

$\boxed{S'}: \quad (E')^2 = m^2$

$$p^{\mu'} = \begin{pmatrix} \gamma & -\gamma v & 0 & 0 \\ -\gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} E \\ P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} 5/4 & -3/4 & 0 & 0 \\ -3/4 & 5/4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{5m}{4} \\ 3m/4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} m \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

τετραορμη ενος σωματιδιου στο συστημα ηρεμιας του

Ασκηση 2



Αν S' = συστημα ηρεμιας του 2

a) E_i, $\vec{P}_i$, T_i στα S, S'

β) η ταχυτητα του 1 στο S' (συστημα ηρεμιας του 2)

$\cos 60 = \frac{1}{2} \qquad \sin 60 = \sqrt{3}/2$

$\vec{v}_1 = \frac{3}{5}(\cos\theta, \sin\theta, 0) = \left(\frac{3}{10}, \frac{3\sqrt{3}}{10}, 0\right)$ | $\gamma_1 = \frac{5}{4}$ | $P_1^\mu = (\overset{E_1}{m\gamma_1}, \overset{\vec{P}_1}{m\gamma_1 \vec{v}_1})$

$\vec{v}_2 = \left(\frac{4}{5}, 0, 0\right)$ | $\gamma_2 = \frac{5}{3}$ | $P_2^\mu = (\overset{E_2}{m\gamma_2}, \overset{\vec{P}_2}{m\gamma_2 \vec{v}_2})$

$$P_1^\mu = m\begin{pmatrix} 5/4 \\ 3/8 \\ 3\sqrt{3}/8 \\ 0 \end{pmatrix} \qquad P_2^\mu = m\begin{pmatrix} 5/3 \\ 4/3 \\ 0 \\ 0 \end{pmatrix} \qquad \Lambda = \begin{pmatrix} 5/3 & -4/3 & 0 & 0 \\ -4/3 & 5/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

για $m = 1$ GeV

$$P_1^2 = -\frac{25}{16} + \frac{9}{64} + \frac{27}{64} = -\frac{100}{64} + \frac{36}{64} = -1 \qquad P_2^2 = -\frac{25}{9} + \frac{16}{9} = -1 \quad [\mathrm{GeV}^2]$$

$$P_2^{\mu'} = m\begin{pmatrix} 5/3 & -4/3 & 0 & 0 \\ -4/3 & 5/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 5/3 \\ 4/3 \\ 0 \\ 0 \end{pmatrix} = m\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\vec{v}_i' = \dfrac{\vec{P}_i'}{E_i}$

$i = 1, 2$

$T_i' = E_i' - m$

$T_1' = \frac{19m}{12} - m = \frac{7m}{12}$

$T_2' = 0$

"ωμος"

$$P_1^{\mu'} = m\begin{pmatrix} 5/3 & -4/3 & 0 & 0 \\ -4/3 & 5/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 5/4 \\ 3/8 \\ 3\sqrt{3}/8 \\ 0 \end{pmatrix} = m\begin{pmatrix} 19/12 \\ -25/24 \\ 3\sqrt{3}/8 \\ 0 \end{pmatrix}$$

$\vec{v}_1' = \dfrac{\vec{P}_1'}{E_1'} = \dfrac{[-25, 9\sqrt{3}, 0]^T}{38}$

$|\vec{v}_1'| \simeq 0.78 \qquad \vec{v}_{12}' = \vec{v}_1'$

$E_1' = 19/12\, m \qquad E_1 = 5/4\, m \qquad$ διατηρηση της ενεργειας ?

"κομψος"

$P_1^\mu \cdot P_{2\mu} = P_1^{\mu'} P_{2\mu'} = \mathbb{P}_1 \cdot \mathbb{P}_2 =$ αναλλοιωτο Lorentz

$$P_1^\mu \cdot P_{2\mu} = m^2\left[-\frac{5}{4}\cdot\frac{5}{3} + \frac{3}{8}\,\frac{4}{3}\right] = m^2\left[-\frac{25}{12} + \frac{6}{12}\right] = -m^2\,\frac{19}{12}$$

$\gamma_{12} = 19/12$

$= \dfrac{1}{\sqrt{1 - v_{12}^2}}$

$\boxed{\text{ξερω οτι στο } S'}$ $\quad P_2^{\mu'} = m(1, 0, 0, 0)$, $\quad P_{1\mu'} = m(-\gamma_{12}, \gamma_{12}\vec{v}_{12})$ $\quad$ αρα $\rightarrow P_1^{\mu'} \cdot P_{2\mu'} = -m^2 \gamma_{12}$

$v_{12} = \sqrt{(1 - \gamma^{-2})} \simeq 0.78$

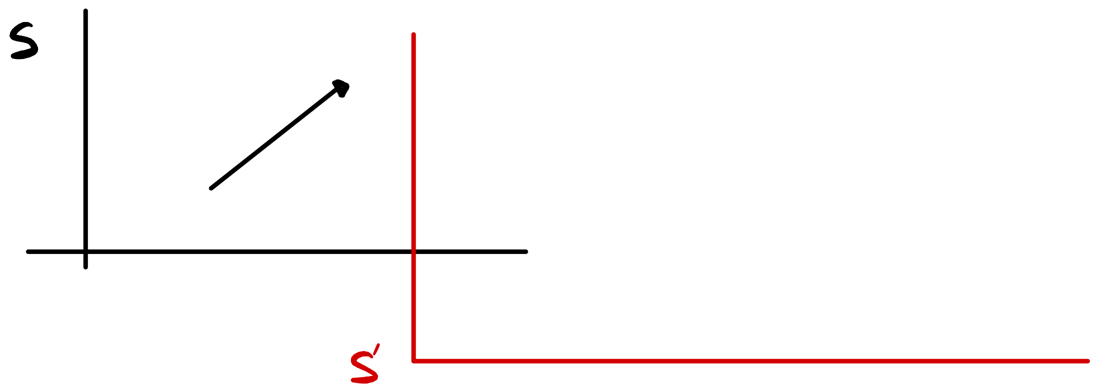
$\mathbb{P}_1 \cdot \mathbb{P}_2 \qquad \mathbb{P}_1^2 \qquad \mathbb{P}_2^2$

S	S'
$\vec{u}_1$	$\vec{u}_1'$
$= \left(\frac{3}{10}, \frac{3\sqrt{3}}{10}, 0\right)$	$(?, ?, 0)$

"ασκησηος"

μετασχηματισμος $u=\left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) \longrightarrow u'=\left(\frac{dx'}{dt'}, \frac{dy'}{dt'}, \frac{dz'}{dt'}\right)$

αφου η προωθηση ειναι // $\hat{x}$ περιμενουμε το $u_y' = u_y$?



$$dx' = \gamma(dx - u\,dt) \qquad dy_1' = dy_1 \qquad dz_1' = dz_1$$
$$dt' = \gamma(dt - u\,dx)$$

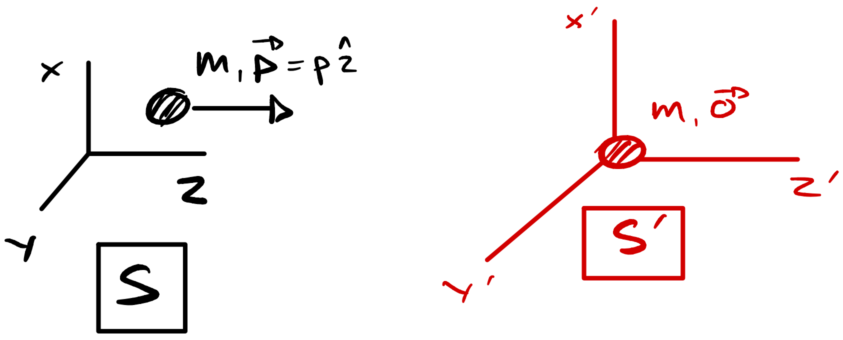
$$\frac{dx_1'}{dt_1'} = \frac{\gamma(dx_1 - u\,dt_1)}{\gamma(dt_1 - u\,dx_1)} = \frac{u_{1x} - u}{1 - u_{1x}\cdot u} = \frac{3/10 - 4/5}{1 - 3/10 \cdot 4/5} = -\frac{25}{38} = u_{1x}'$$

$$\frac{dy_1'}{dt_1'} = \frac{dy_1}{\gamma(dt_1 - u\,dx_1)} = \frac{u_{1y}/\gamma}{1 - u_{1x}\cdot u} = \frac{3\sqrt{3}/10 \cdot \frac{3}{5}}{38/50} = \frac{9\sqrt{3}}{38} = u_{1y}'$$

$$\frac{dz_1'}{dt_1'} = \frac{dz_1}{\gamma(dt_1 - u\,dx_1)} = \emptyset$$

$$\boxed{\begin{array}{c} \vec{u}_1 = \left(\frac{3}{10}, \frac{3\sqrt{3}}{10}, 0\right) \\ \gamma = 5/3 \quad u_1 = 4/5 \\ S \end{array}} \qquad \boxed{\begin{array}{c} \vec{u}_1' = \left(-\frac{25}{38}, \frac{9\sqrt{3}}{38}, 0\right) \\ S' \end{array}}$$

Προώθηση προς το σύστημα ηρεμίας σωματιδίου



$\boxed{S}$: $p^{\mu} = (E, 0, 0, p)$

$\boxed{S'}$: $p^{\mu'} = (m, 0, 0, 0)$

$$\boxed{\vec{u} = \frac{\vec{p}}{E}} \qquad \boxed{\gamma = \frac{E}{m}}$$

$$u = u_x = p/E \qquad \gamma = E/m$$

$$\begin{pmatrix} \gamma & 0 & 0 & -\gamma u \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma u & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} E \\ 0 \\ 0 \\ p \end{pmatrix} = \begin{bmatrix} \gamma E - \gamma u p \\ 0 \\ 0 \\ -\gamma u E + \gamma p \end{bmatrix}$$

$$= \begin{bmatrix} E/m \cdot E - \frac{E}{m} \cdot \frac{p}{E} \cdot p \\ 0 \\ 0 \\ -\frac{E}{m} \cdot \frac{p}{E} \cdot E + \frac{E}{m} \cdot p \end{bmatrix} = \begin{bmatrix} \frac{E^2 - p^2}{m} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} m \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

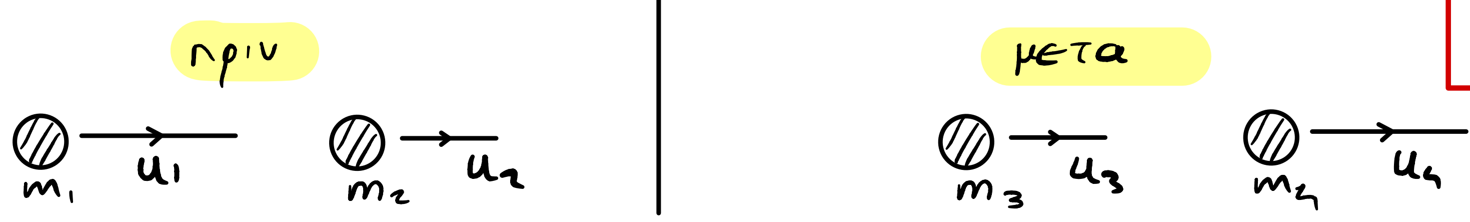
$$E^2 = p^2 + m^2$$

Διατηρηση 3-ορμης & μετ. Lorentz

σε 1-D κρουση

κρουση σε 1 διασταση ($\vec{u}_1 // \vec{u}_2 // \vec{u}_3 // \vec{u}_4$)

σημειωση: στην διαρκεια της διαλεξης εδεσα $m_1 = m_2 = m_3 = m_4 = m$ το οποιο δεν ηταν απαραιτητο



κ.Μ. ($\vec{p}_i = m_i \vec{u}_i$)

$$\boxed{S} \qquad m_1 u_1 + m_2 u_2 \overset{?}{=} m_3 u_3 + m_4 u_4$$

$$\boxed{S'} \qquad m_1 u_1' + m_2 u_2' \overset{?}{=} m_3 u_3' + m_4 u_4'$$

Μετ. Lorentz

$$u_i' = \frac{u_i - v}{1 - u_i v}$$

$$m_1 \frac{u_1 - v}{1 - u_1 v} + m_2 \frac{u_2 - v}{1 - u_2 v} \overset{?}{=} m_3 \frac{u_3 - v}{1 - u_3 v} + m_4 \frac{u_4 - v}{1 - u_4 v}$$

Σ.Μ. ($\vec{p}_i = \gamma_i m_i \vec{u}_i$)

$$p_1' = \gamma_1' m u_1' = \frac{1}{\sqrt{1 - (u_1')^2}} m_1 \cdot \frac{u_1 - v}{1 - u_1 v}$$

$$= \frac{1}{\sqrt{1 - \left(\frac{u_1 - v}{1 - u_1 v}\right)^2}} m_1 \cdot \frac{u_1 - v}{1 - u_1 v}$$

$$= \frac{1}{\sqrt{\frac{1 - 2u_1 \cdot v + u_1^2 v^2 - u_1^2 - v^2 + 2u_1 v}{1 - 2u_1 \cdot v + u_1^2 v^2}}} m_1 \cdot \frac{u_1 - v}{1 - u_1 v}$$

$$= \frac{\cancel{1 - u_1 v}}{\left(1 \cancel{- 2u_1 \cdot v} + u_1^2 v^2 - u_1^2 - v^2 \cancel{+ 2u_1 v}\right)^{1/2}} m_1 \cdot \frac{u_1 - v}{\cancel{1 - u_1 v}}$$

$$= \frac{1}{(1 + u_1^2 \cdot v^2 - u_1^2 - v^2)^{1/2}} \cdot m_1 (u_1 - v)$$

$$= \frac{1}{\sqrt{1 - u_1^2}} \frac{1}{\sqrt{1 - v^2}} m_1 (u_1 - v)$$

$$p_1' = \underbrace{\frac{1}{\sqrt{1 - u_1^2}}}_{\gamma_1} \underbrace{\frac{1}{\sqrt{1 - v^2}}}_{\gamma} m_1 (u_1 - v) = \gamma(\gamma_1 m_1 u_1 - v \gamma_1 m_1)$$

πριν $\qquad\qquad\qquad\qquad$ μετα

$$\underbrace{\frac{m_1 u_1}{\sqrt{1-u_1^2}}}_{P_1} + \underbrace{\frac{m_2 u_2}{\sqrt{1-u_2^2}}}_{P_2} = \underbrace{\frac{m_3 u_3}{\sqrt{1-u_3^2}}}_{P_3} + \underbrace{\frac{m_4 u_4}{\sqrt{1-u_4^2}}}_{P_4} \qquad P_1+P_2=P_3+P_4$$

$$\Big\downarrow \text{LT}$$

$$\frac{m_1 u_1'}{\sqrt{1-u_1'^2}} + \frac{m_2 u_2'}{\sqrt{1-u_2'^2}} = \frac{m_3 u_3'}{\sqrt{1-u_3'^2}} + \frac{m_4 u_4'}{\sqrt{1-u_4'^2}}$$

$$\downarrow$$

$$\gamma(\gamma_1 m_1 u_1 - \upsilon\gamma_1 m_1) + \gamma(\gamma_2 m_2 u_2 - \upsilon\gamma_2 m_2) = \gamma(\gamma_3 m_3 u_3 - \upsilon\gamma_3 m_3) + \gamma(\gamma_4 m_4 u_4 - \upsilon\gamma_4 m_4)$$

$$(\underbrace{\gamma_1 m_1 u_1}_{P_1} - \upsilon\gamma_1 m_1) + (\underbrace{\gamma_2 m_2 u_2}_{P_2} - \upsilon\gamma_2 m_2) = (\underbrace{\gamma_3 m_3 u_3}_{P_3} - \upsilon\gamma_3 m_3) + (\underbrace{\gamma_4 m_4 u_4}_{P_4} - \upsilon\gamma_4 m_4)$$

$$-\upsilon\gamma_1 m_1 - \upsilon\gamma_2 m_2 = -\upsilon\gamma_3 m_3 - \upsilon\gamma_4 m_4$$

$$\upsilon(\gamma_1 m_1 + \gamma_2 m_2) = \upsilon(\gamma_3 m_3 + \gamma_4 m_4)$$

$$\upsilon(E_1 + E_2) = \upsilon(E_3 + E_4)$$

δηλαδη δειξαμε οτι :

Αν $\quad P_1 + P_2 = P_3 + P_4 \quad$ στο $\boxed{S}$

τοτε $\quad P_1' + P_2' = P_3' + P_4' \quad$ στο $\boxed{S'}$

εφοσον $\quad E_1 + E_2 = E_3 + E_4 \quad$ και $\quad \vec{P}_i \equiv \gamma_i m \vec{u}_i \quad E_i \equiv \gamma_i m$

Πιο συντομη αποδειξη : χρησιμοποιω οτι $P_i' = \gamma(P_i - \upsilon E)$

$$\underbrace{\gamma(P_1 - \upsilon E_1)}_{P_1'} + \underbrace{\gamma(P_2 - \upsilon E_2)}_{P_2'} = \underbrace{\gamma(P_3 - \upsilon E_3)}_{P_3'} + \underbrace{\gamma(P_4 - \upsilon E_4)}_{P_4'}$$

$$(P_1 + P_2) - \upsilon(E_1 + E_2) = (P_3 + P_4) - \upsilon(E_3 + E_4)$$

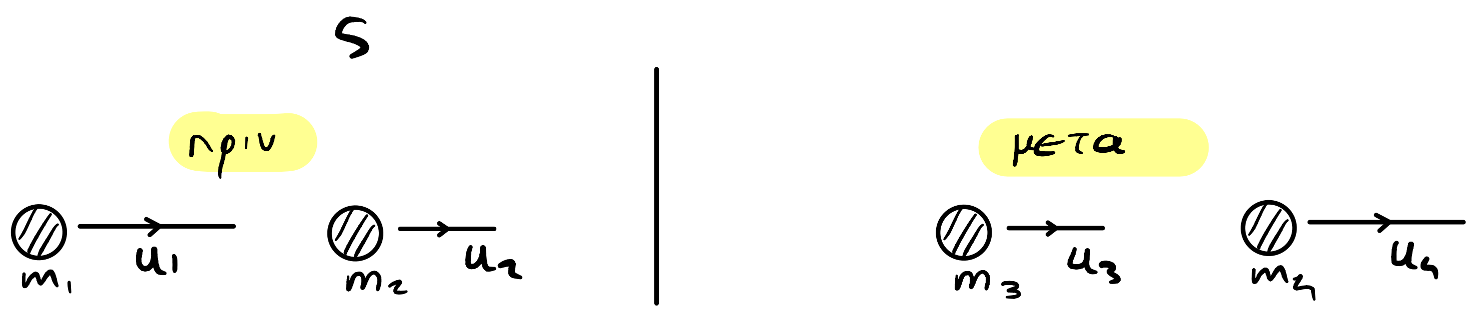
για καθε $\quad 0 \leq \upsilon < 1$

Διατήρηση ορμής στην ΚΜ

8η διάλεξη 15.12.21

& μετ. Γαλιλαίου

κρούση σε 1 διάσταση ($\vec{u}_1 // \vec{u}_2 // \vec{u}_3 // \vec{u}_4$)



$\boxed{S}$ $$m_1u_1 + m_2u_2 = m_3u_3 + m_4u_4 \qquad (\text{Κ.Ν.})$$

$$u_i' = u_i - v \qquad (\text{μετ. Γαλιλαίου})$$

$\boxed{S'}$ $$m_1u_1' + m_2u_2' = m_3u_3' + m_4u_4'$$

$$m_1(u_1-v) + m_2(u_2-v) = m_3(u_3-v) + m_4(u_4-v)$$

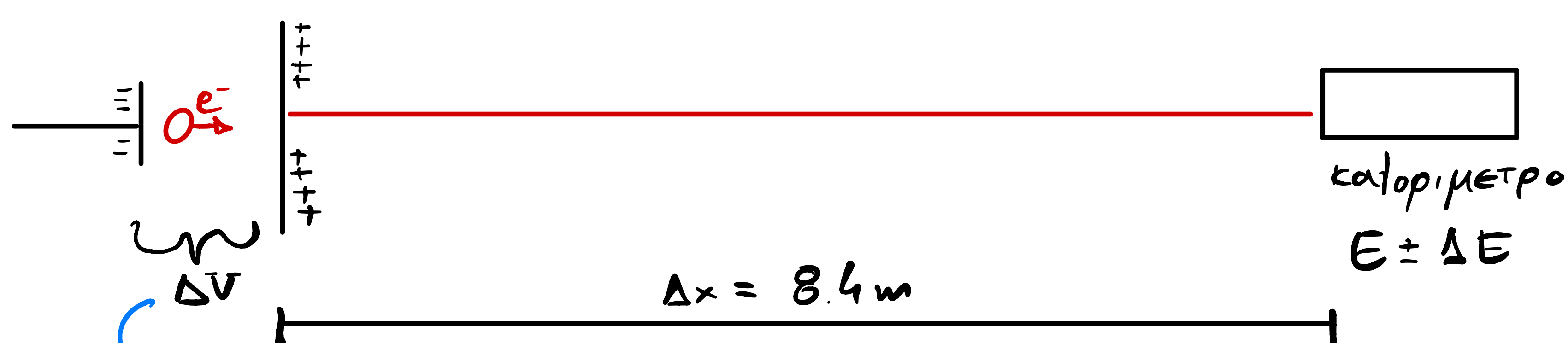
$$m_1u_1 + m_2u_2 - v(m_1+m_2) = m_3u_3 + m_4u_4 - v(m_3+m_4)$$

για να ισχύει πρέπει $m_1 + m_2 = m_3 + m_4$

Έλεγχος του $\frac{1}{2}mv^2$

W. Bertozzi (1962)

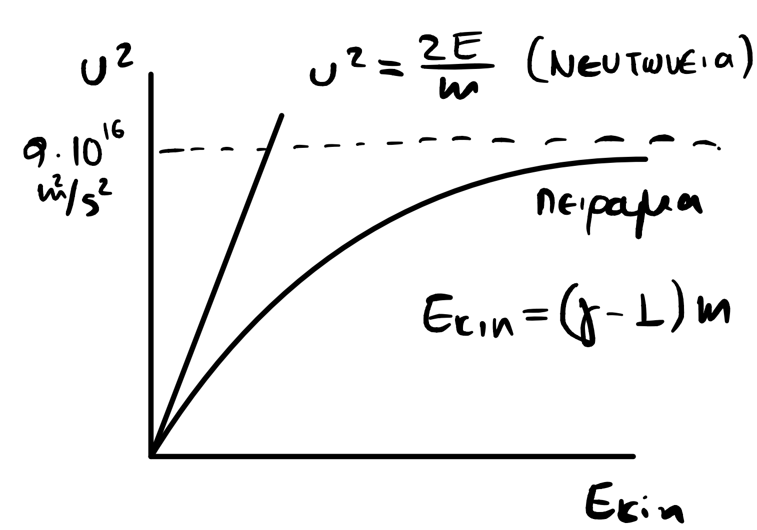
$E_{kin}^{KM} = \frac{1}{2}mv^2$ $\qquad e^-$ $\qquad m_{e^-} = 511$ keV



$E_{kin} = q \cdot \Delta V$ $\qquad \Delta t$ $\qquad v = \frac{\Delta x}{\Delta t}$

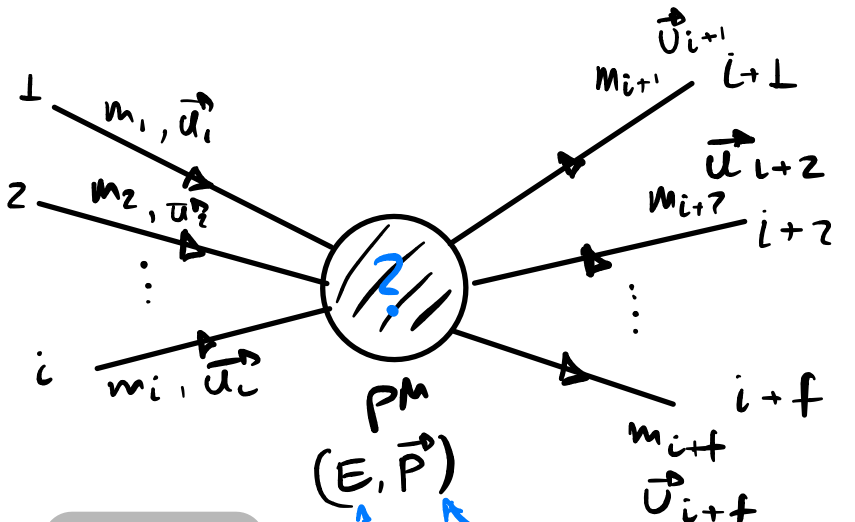
E_{kin} [MeV]	Δt [ns]	v [$10^8 \frac{m}{s}$]
0.5	32.3	2.6
1.0	30.8	2.73
1.5	29.2	2.88
4.5	28.4	2.99
15	28.0	3.00

$\stackrel{?}{=} \frac{1}{2}mv^2$



"Youtube Bertozzi ultimate speed"

Τετραορμή συστήματος σωματιδίων



$i \to f$

Κ.Μ.

$$\begin{cases} m_1 + m_2 + \dots m_i = m_{i+1} + m_{i+2} + \dots \quad m_{i+f} \\ m_1\vec{u}_1 + m_2\vec{u}_2 + \dots m_i\vec{u}_i = m_{i+1}\vec{u}_{i+1} + m_{i+2}\vec{u}_{i+2} + \dots \quad m_{i+f}\vec{u}_{i+f} \end{cases}$$

$$\sum_{n=1}^{i} m_n = \sum_{n=i+1}^{i+f} m_n \qquad (1 \text{ εξ.})$$

$$\sum_{n=1}^{i} m_n\vec{u}_n = \sum_{n=i+1}^{i+f} m_n\vec{u}_n \qquad (3 \text{ εξ.})$$

ΣΜ

$$\sum_{n=1}^{i} P_n^\mu = \sum_{n=i+1}^{i+f} P_n^\mu = P^\mu = P^\mu_{cm} \qquad (4 \text{ εξ.})$$

$$\begin{cases} \sum_{n=1}^{i} \underbrace{\gamma_n m_n}_{E_n} = \sum_{n=i+1}^{i+f} \underbrace{\gamma_n m_n}_{E_n} \\ \sum_{n=1}^{i} \underbrace{\gamma_n m_n \vec{u}_n}_{\vec{P}_n} = \sum_{n=i+1}^{i+f} \underbrace{\gamma_n m_n \vec{u}_n}_{\vec{P}_n} \end{cases}$$

> Η συνολική τετραορμή ενός κλειστού συστήματος είναι σταθερή

$$P^\mu = \sum_n P_n^\mu = (E, \vec{P})$$

M = αναλλοίωτη μάζα συστήματος $\equiv \sqrt{s} = \sqrt{E^2 - (\vec{P})^2}$

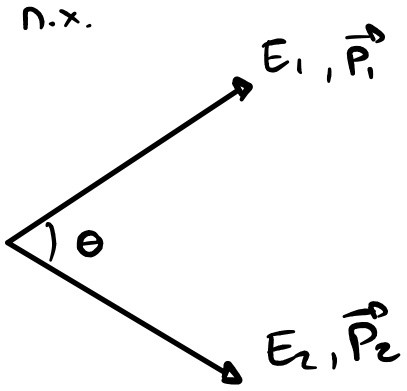
π.χ. έχουμε $2 \to 2$

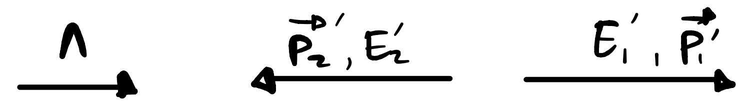
$$M \neq m_1 + m_2$$

Σύστημα κέντρου ορμής (κέντρο μάζας) $\equiv \vec{P} = 0$

$$\underset{\boxed{S}}{P^\mu_{cm} \equiv P^\mu = (E, \vec{P})} \xrightarrow{\Lambda} \underset{\boxed{S'}}{P^{\mu'} = (M, \vec{0})}$$

Βοηθάει να σκεφτόμαστε το P^μ σαν ένα υποθετικό σωματίδιο "CM" το οποίο "κουβαλάει" την ενέργεια & ορμή ολόκληρου του συστήματος π.χ.





$$\vec{P}_1' = -\vec{P}_2' = \vec{P}^*$$
$$|\vec{P}_1'| = |\vec{P}_2'| = |\vec{P}^*|$$

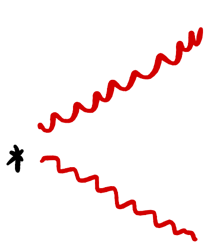
$\boxed{S}$ $\boxed{S'}$

$p^\mu = (E, \vec{p})$ $\Lambda(\vec{u}_{cm}) = ?$ $p^{\mu'} = (M, \vec{0})$

$$\vec{u}_{cm} = \frac{\vec{p}}{E} = \frac{\vec{p}_1 + \vec{p}_2}{E_1 + E_2}$$

$$\gamma = \frac{E}{M} = \frac{E_1 + E_2}{\sqrt{(E_1+E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2}}$$

Μαζα συστηματος 2γ



$\vec{p}_1 = (3,4,0)$ [GeV]

$\vec{p}_2 = (3,-4,0)$ [GeV]

μαζα συστηματος ?

$$M = \begin{cases} 0 & (A) \\ >0 & (B) \\ \text{εξαρταται} & (C) \end{cases}$$

$\boxed{S}$

$$E_1^2 = \cancel{m_1^2}^{\,0} + (\vec{p}_1)^2$$

$$p_1^\mu = (E_1, \vec{p}_1) = (|\vec{p}_1|, \vec{p}_1) = (5, 3, 4, 0)$$
$$p_2^\mu = (E_2, \vec{p}_2) = (|\vec{p}_2|, \vec{p}_2) = (5, 3, -4, 0)$$

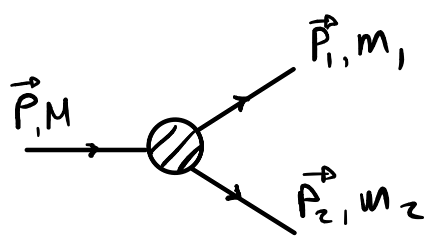
$$p^\mu = (E_1 + E_2, \vec{p}_1 + \vec{p}_2) = (10, 6, 0, 0)$$

$$p^\mu p_\mu = -M^2 = -10^2 + 6^2 = 64 \quad [\text{GeV}]$$
$$M = 8 \; [\text{GeV}]$$

$1 \to 2$ γενική περίπτωση

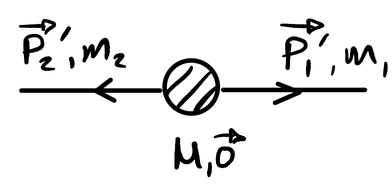
9η διάλεξη 21.12.21

$M \to m_1\, m_2$



$\boxed{S}$

κέντρο μάζας (ορμής)



$\vec{P}_1' = -\vec{P}_2' = \vec{P}^*$

$\boxed{S'}$

$P^\mu = P_1^\mu + P_2^\mu \quad \boxed{S}$

$P^{\mu'} = P_1^{\mu'} + P_2^{\mu'} \quad \boxed{S'}$

αλλά $\quad P^\mu \neq P^{\mu'} = \Lambda^{\mu'}_{\ \mu} P^\mu$

όμως $\quad (P^\mu)^2 = (P^{\mu'})^2$

π.χ.

$P^\mu = (E, \vec{P})$
$P_1^\mu = (E_1, \vec{P}_1)$
$P_2^\mu = (E_2, \vec{P}_2)$
$\boxed{S}$

$P^{\mu'} = (M, \vec{0})$
$P_1^{\mu'} = (E_1^*, \vec{P}^*)$
$P_2^{\mu'} = (E_2^*, -\vec{P}^*)$
$\boxed{S'}$

$P^{\mu'} \neq P_1^\mu + P_2^\mu$ όμως $(P')^2 = (P_1 + P_2)^2$

$s = M^2 = (E_1 + E_2)^2 - (\vec{P}_1 + \vec{P}_2)^2$

$\sqrt{s} \equiv M$

αναλλοίωτα Lorentz

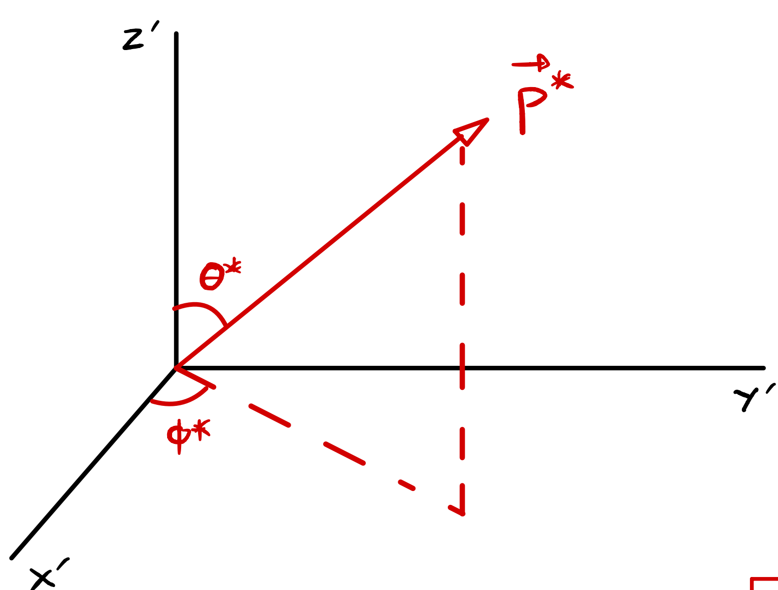
βαθμοί ελευθερίας στο Κ.Ο. για δεδομένα M, m_1, m_2 ?

$P^{\mu'} = P_1^{\mu'} + P_2^{\mu'} \longrightarrow$

$3 \times 4 = 12$ παράμετροι
-4 εξισώσεις (δεσμοί)
-4 P^μ γνωστό στο Κ.Ο.
-2 $P_1^2 = m_1^2 \quad P_2^2 = m_2^2 \quad (+ - - -)$

2 βαθμοί ελευθερίας

θ^*, ϕ^*



$P' = (M, \vec{0})$
$P_1' = (E_1^*, \vec{P}^*)$
$P_2' = (E_2^*, -\vec{P}^*)$

$\vec{P}^* = (p^* \sin\theta^* \cos\phi^*, p^* \sin\theta^* \sin\phi^*, p^* \cos\theta^*)$

πλήρως προσδιορίσιμα συναρτήσει των μαζών

$$\boxed{E_1^* = \frac{M^2 + m_1^2 - m_2^2}{2M}, \quad E_2^* = \frac{M^2 + m_2^2 - m_1^2}{2M}}$$

$$\boxed{|\vec{P}^*_{1,2}|^2 = \frac{[M^2 - (m_1 - m_2)^2][M^2 - (m_1 + m_2)^2]}{4M^2} = |\vec{P}^*|^2}$$

$\theta^* = \phi^* = 0$

$P^{\mu'} = (M, 0, 0, 0)$

$P_1^{\mu'} = (E_1^*, 0, 0, p^*)$

$P_2^{\mu'} = (E_2^*, 0, 0, -p^*)$

$P' = P_1' + P_2' \rightsquigarrow (P' - P_1')^2 = (P_2')^2$

$(+ - - -)$

$(P')^2 + (P_1')^2 - 2P' \cdot P_1' = m_2^2$

$M^2 + m_1^2 - 2ME_1^* = m_2^2$

$$E_1^* = \frac{M^2 + m_1^2 - m_2^2}{2M}$$

ομοίως $\boxed{E_2^* = \dfrac{M^2 + m_2^2 - m_1^2}{2M}}$

$\vec{P}_1^* \equiv \vec{P}^*$

$p \equiv |\vec{p}|$

$(E_1^*)^2 = m_1^2 + |\vec{P}_1^*|^2 = m_1^2 + (p^*)^2$

$(p^*)^2 = (E_1^*)^2 - m_1^2 = (E_2^*)^2 - m_2^2$

$A = m_1 - m_2 \qquad B = m_1 + m_2 \qquad A^2 + B^2 = (m_1^2 + m_2^2)2$

$$4M^2 (p^*)^2 = \left(M^2 + (m_1 - m_2)(m_1 + m_2)\right)^2 - m_1^2 4M^2 \qquad (1)$$

$$= \left(M^2 - (m_1 - m_2)(m_1 + m_2)\right)^2 - m_2^2 4M^2 \qquad (2)$$

$(1) \to \quad = M^4 + (A \cdot B)^2 + 2A \cdot B M^2 - m_1^2 4M^2$

$(2) \to \quad = M^4 + (A \cdot B)^2 - 2A \cdot B M^2 - m_2^2 4M^2$

$\frac{1}{2}((1)+(2)) \quad = M^4 + (A \cdot B)^2 - 2(m_1^2 + m_2^2)M^2$

$= M^4 + (A \cdot B)^2 - (A^2 + B^2) M^2$

$= M^2(M^2 - A^2) - B^2(M^2 - A^2)$

$4M^2 (p^*)^2 = (M^2 - B^2)(M^2 - A^2)$

$$(p^*)^2 = |\vec{P}^*_{1,2}|^2 = \frac{[M^2 - (m_1 - m_2)^2]\,[M^2 - (m_1 + m_2)^2]}{4M^2}$$

$2 \to 1$

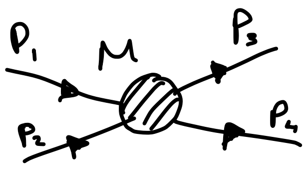
ομοίως με την $1 \to 2$

$P^\mu = p_1^\mu + p_2^\mu \qquad 1 \to 2$

$p_1^\mu + p_2^\mu = P^\mu \qquad 2 \to 1$

2 → 2

$[m_1 + m_2 \xrightarrow{M} m_3 + m_4]$



σκεφτόμαστε το 2→2 σαν 2→1 και 1→2 με ενδιάμεση "κατάσταση" $(M, \vec{P})$

$P = (M, \vec{P})$ για $\vec{P} = 0$: κέντρο μάζας (μηδενική ολική ορμή)

$$p_1 + p_2 = p_3 + p_4$$

$$\boxed{E_1^* = \frac{M^2 + m_1^2 - m_2^2}{2M} , \quad E_2^* = \frac{M^2 + m_2^2 - m_1^2}{2M}} \; *$$

$$\boxed{|\vec{p}^{\,*}_{1,2}|^2 = \frac{[M^2 - (m_1 - m_2)^2]\,[M^2 - (m_1 + m_2)^2]}{4M^2}} \; *$$

για την τελική κατάσταση (p_3, p_4) αντικαθιστούμε $(1,2) \to (3,4)$

π.χ.

$$E_3^* = \frac{M^2 + m_3^2 - m_4^2}{2M}$$

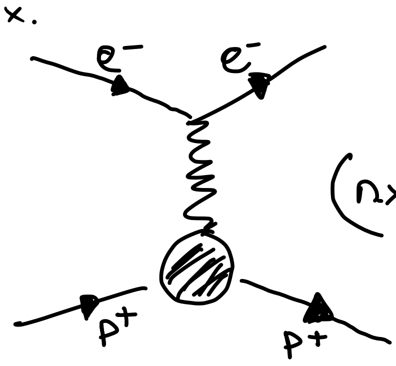
$$|\vec{p}^{\,*}_{3,4}| = \frac{[M^2 - (m_3 - m_4)^2]\,[M^2 - (m_3 + m_4)^2]}{2M}$$

ελαστική κρούση

Νευτώνια : $T_{πριν} = T_{μετα}$ {θερμότητα, ήχος, τριβή}

Σχετικιστικά : $m_1\, m_2 \dots m_n \to m_1\, m_2 \dots m_n$ {ίδιες μάζες πριν + μετά}

π.χ.



(π.χ. $e^- p^+ \to e^- p^+$)

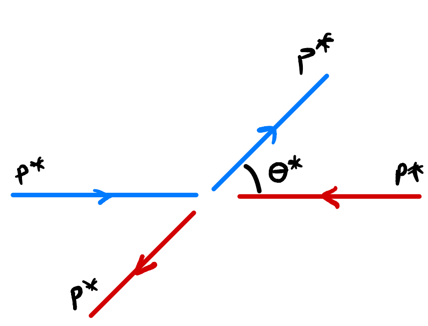
S^*

$E_1^* = E_3^*$ $\quad E_2^* = E_4^*$

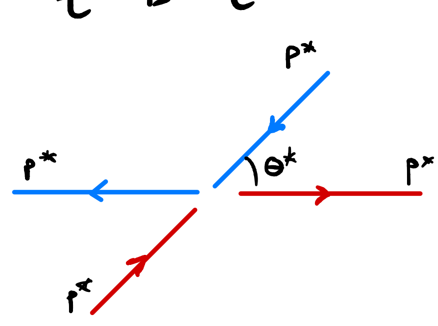
$|\vec{p}^{\,*}_{1,2}| = |\vec{p}^{\,*}_{3,4}| = |\vec{p}^{\,*}|$

$m_1 = m_3$ και $m_2 = m_4$

CM



$t \to -t'$



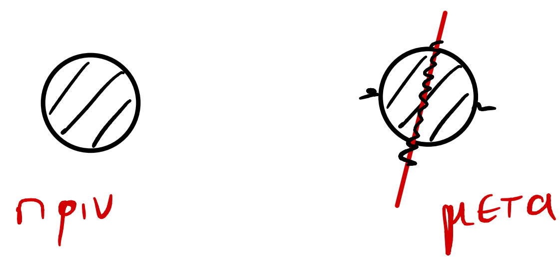
συμμετρία αντιστροφής χρόνου απαιτεί ότι $|\vec{p}_1^{\,*}| = |\vec{p}_2^{\,*}| = |\vec{p}_3^{\,*}| = |\vec{p}_4^{\,*}| = p^*$!

παραδειγματα αντιδρασεων

1 → 2

($M = m_1 + m_2$ νευτωνεια)

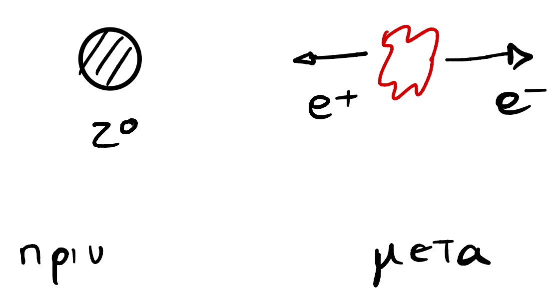
Νευτωνια στο Κ.Ο.



τα δυο κομματια παραμενουν ακινητα διοτι $M = m_1 + m_2$

$Z^0 \to e^- e^+$ $m_Z = 91$ GeV $m_{e^-} = m_{e^+} = 511$ keV

$m_Z/m_e \simeq 178$ k



μαζα των προϊοντων μικροτερη της αρχικης !

$M > m_{e^-} m_{e^+}$

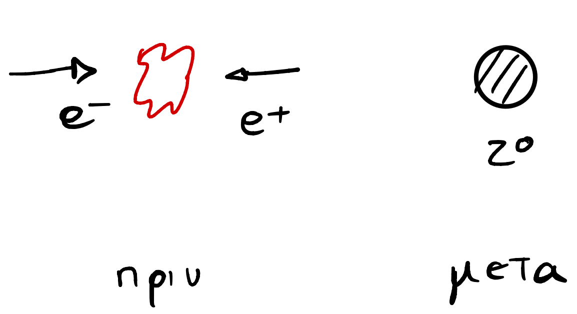
τα $e^\pm$ εχουν μεγαλη ταχυτητα ακομη και στο Κ.Ο.

$p^* \simeq M/2$

μπορει η αλληλεπιδραση να λαβει χωρα με την αντιθετη χρονικη σειρα ?

T: { συμμετρια αντιστροφης χρονου } ~ διατηρηση της ενεργειας (Noether)

2 → 1



συμμετρια αντιστροφης χρονου

$m_{e^-} + m_{e^+} \leq M_Z$

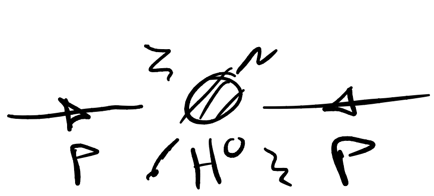
μαζα του προϊοντος ειναι μεγαλυτερη της αρχικης !

CERN

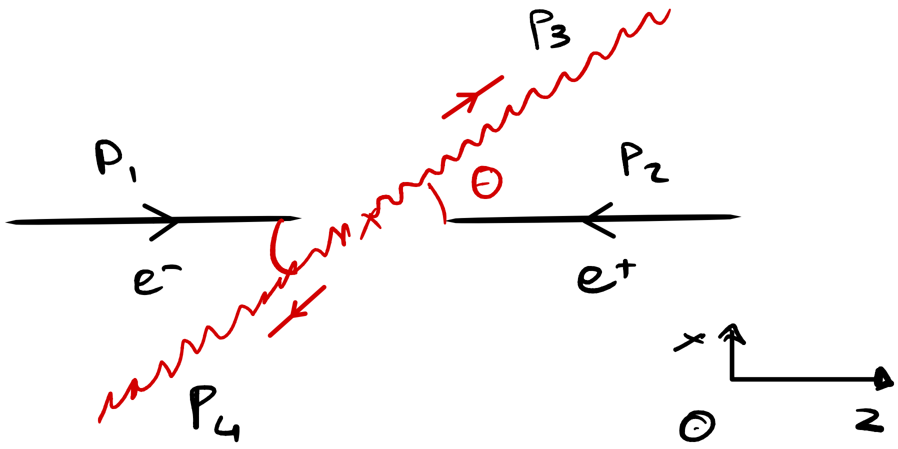
$pp \to H^0 + (\dots)$

Higgs boson

Large Hadron Collider



Εξαυλωση $e^-e^+ \to 2\gamma$



$m_{e^+} + m_{e^-} \to 0 + 0$!

η μαζα των επιμερους συστατικων δεν διατηρειτε πριν // μετα

η αναλλοιωτη μαζα του συστηματος ?

$$\boxed{E_\gamma^{min} = ?}$$

$$P_1 = \left(\sqrt{m_e^2+p^2},\, 0,\, 0,\, p\right) \qquad P_3 = \left(E_\gamma,\, 0,\, E_\gamma \sin\theta,\, E_\gamma \cos\theta\right)$$

$$P_2 = \left(\sqrt{m_e^2+p^2},\, 0,\, 0,\, -p\right) \qquad P_4 = \left(E_\gamma,\, 0,\, -E_\gamma \sin\theta,\, -E_\gamma \cos\theta\right)$$

$$E_\gamma = \sqrt{m_e^2+p^2}$$

$E_\gamma^{min} = m_e = 511$ keV

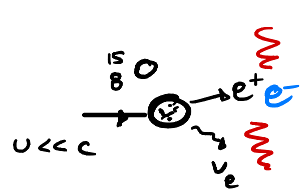
για $p \to 0$

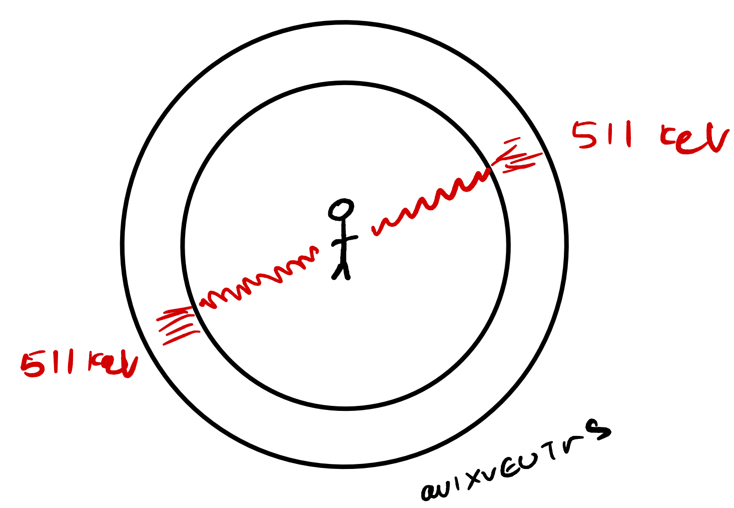
Positron Emission Tomography

ραδιοφαρμακο

$^{15}_{8}O \to {}^{15}_{7}N\, e^+ \nu$

$p \to n\, e^+ \nu$

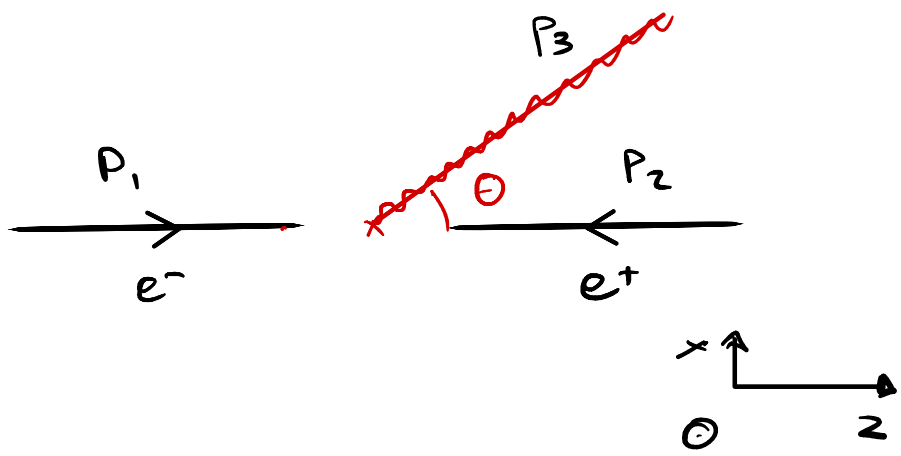




$\Delta\phi \simeq \pi$

$\Delta t \leq 0.5$ ns

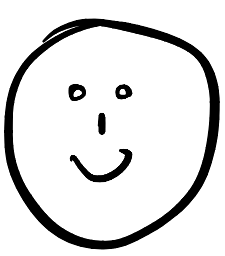
Εξαΰλωση $e^- e^+ \to \gamma$



$$E_\gamma^{min} = ?$$

$$p_1 = \left(\sqrt{m_e^2 + p^2}, 0, 0, p\right) \qquad p_3 = \left(E_\gamma, 0, E_\gamma \sin\theta, E_\gamma \cos\theta\right)$$

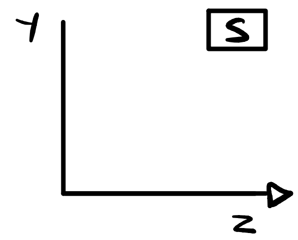
$$p_2 = \left(\sqrt{m_e^2 + p^2}, 0, 0, -p\right) \qquad p_4 = \left(0, 0, 0, 0\right)$$

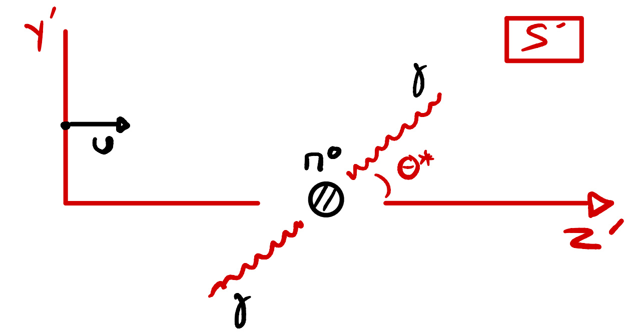


$$p_1 + p_2 \overset{?}{=} p_3 + p_4$$

$E'_\gamma = E^*_\gamma = m/2$

προβλημα 2 (2021)





$$\Lambda(-\vec{v}) = \begin{bmatrix} \gamma & 0 & 0 & \gamma u \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma u & 0 & 0 & \gamma \end{bmatrix}$$

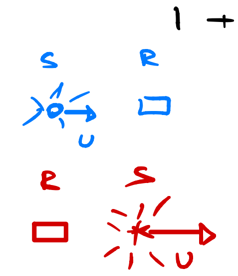
$$m\begin{bmatrix}1\\0\\0\\0\end{bmatrix} = \frac{m}{2}\begin{bmatrix}1\\ \sin\theta^*\\0\\ \cos\theta^*\end{bmatrix} + \frac{m}{2}\begin{bmatrix}1\\ -\sin\theta^*\\0\\ -\cos\theta^*\end{bmatrix}$$ $\boxed{S'}$

$$\begin{bmatrix} \gamma & 0 & 0 & \gamma u \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma u & 0 & 0 & \gamma \end{bmatrix} \frac{m}{2}\begin{bmatrix}1\\ \sin\theta^*\\0\\ \cos\theta^*\end{bmatrix} = \begin{bmatrix} \frac{\gamma m}{2}(1+u\cos\theta^*) \\ \frac{m}{2}\sin\theta^* \\ 0 \\ \frac{\gamma m}{2}(u+\cos\theta^*) \end{bmatrix} = E\begin{bmatrix}1\\ \sin\theta\\0\\ \cos\theta\end{bmatrix}$$

$E = \frac{\gamma m}{2}(1+u\cos\theta^*)$ | $\sin\theta = \dfrac{\sin\theta^*}{\gamma(1+u\cos\theta^*)}$ $\qquad \cos\theta = \dfrac{u+\cos\theta^*}{1+u\cos\theta^*}$

$$\boxed{f = \gamma(1+u\cos\theta^*)\, f^*}$$

$\boxed{\text{διαφορες Doppler}}$



blue shift
κυανη μετατοπιση

red shift
ερυθρη μετατοπιση

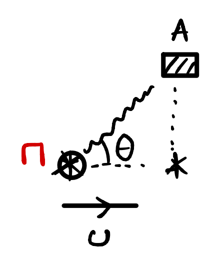
$\theta^* = 0$ $\quad f = \sqrt{\dfrac{1+u}{1-u}}\, f^*$ | $\theta = 0$

$\theta^* = \frac{\pi}{2}$ $\quad f = \gamma f^*$ Blue shift

$\cos\theta = u$

↖ συχνοτητα στον ανιχνευτη για ακτινα με $\theta^* = \pi/2$ στο $\boxed{S'}$

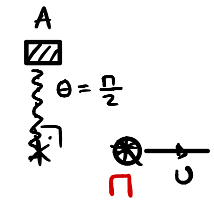
#1



Πηγη Ανιχνευτης

$\theta = \pi/2$ $\rightsquigarrow$ $\cos\theta^* = -u$

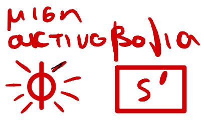
#2



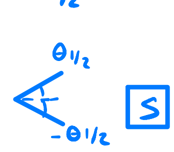
$$f = \gamma(1-u^2)f^* = \sqrt{1-u^2}\, f^*$$

$f = f^*/\gamma$ red shift $\rightsquigarrow$ συχνοτητα στον ανιχνευτη για ακτινα με $\theta = \pi/2$ στο $\boxed{S}$

$\theta^* = \pm\frac{\pi}{2}$
μιση ακτινοβολια



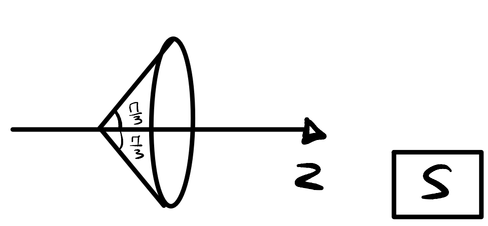
$\theta_{1/2}$



$\sin\theta_{1/2} = \frac{1}{\gamma}$ $\qquad \cos\theta_{1/2} = u$

$\tan(\theta_{1/2}) = (\gamma u)^{-1}$

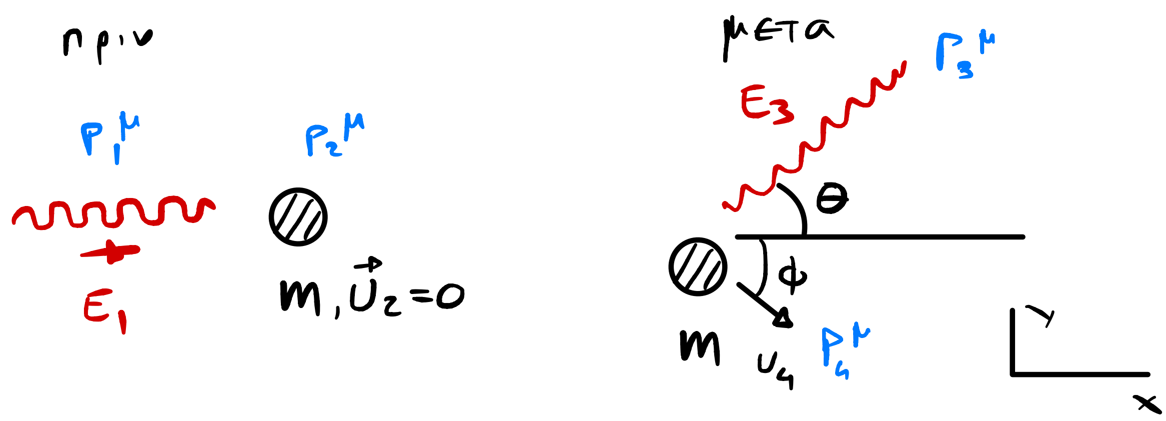
π.χ. για $u = \frac{1}{2}$ $\qquad \theta_{1/2} = 60°$



Τι χρωμα εχει ενα πρωτονιο

Διαλεξη 10η 22.12.2021

σκεδαση Compton



m = γνωστο

για E_1 = γνωστο

$E_3 = E_3(\theta, E_1) = ?$

$\lambda_3 = \lambda_3(\theta, \lambda_1) = ?$

$E = h\nu = h\frac{c}{\lambda}$

$\nu \cdot \lambda = c$

$\lambda = \frac{hc}{E}$

$\frac{1}{E} = \lambda/hc$

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu$$

$$\begin{bmatrix} E_1 \\ E_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} m \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} E_3 \\ E_3\cos\theta \\ E_3\sin\theta \\ 0 \end{bmatrix} + \begin{bmatrix} E_4 \\ p_4\cos\phi \\ -p_4\sin\phi \\ 0 \end{bmatrix}$$

$$\begin{array}{ccccccc} 1 & & 2 & & 3 & & 4 \\ \begin{bmatrix} E_1 \\ E_1 \\ 0 \\ 0 \end{bmatrix} & + & \begin{bmatrix} m \\ 0 \\ 0 \\ 0 \end{bmatrix} & - & \begin{bmatrix} E_3 \\ E_3\cos\theta \\ E_3\sin\theta \\ 0 \end{bmatrix} & = & \begin{bmatrix} E_4 \\ p_4\cos\phi \\ -p_4\sin\phi \\ 0 \end{bmatrix} \end{array}$$

$$(p_1^\mu + p_2^\mu - p_3^\mu)^2 = (p_4^\mu)^2$$

$(- + + +) \to p^2 = -m^2$

$(+ - - -)$

$(p^\mu)^2 = m^2$

$p_1^\mu \cdot p_{2\mu} = E_1 \cdot E_2 - \vec{p}_1 \cdot \vec{p}_2$

$$(p_1^\mu)^2 + (p_2^\mu)^2 + (p_3^\mu)^2 + 2p_1^\mu p_{2\mu} - 2p_1^\mu p_{3\mu} - 2p_2^\mu p_{3\mu} = m^2$$

$$0 + m^2 + 0 + 2E_1 m - 2(E_1E_3 - E_1E_3\cos\theta) - 2mE_3 = m^2$$

$$\frac{1}{mE_1E_3}\left(E_1 m - E_1E_3(1-\cos\theta) - mE_3\right) = 0$$

$$\frac{1}{E_3} - \frac{1}{m}(1-\cos\theta) - \frac{1}{E_1} = 0$$

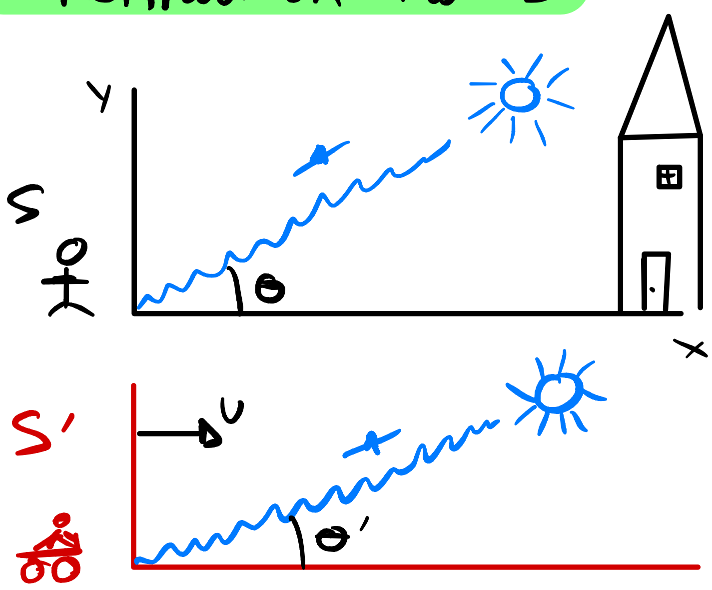
$$\lambda_3 = \lambda_1 + \frac{hc}{m}(1-\cos\theta)$$

$\lambda_c = \frac{hc}{m}$ = μηκος κυματος Compton

= μηκος κυματος ενος φωτονιου ενεργειας $E = m$

για ενα πρωτονιο $\lambda_c \simeq 1.3$ fm

Αποπλάνηση φωτός



για εισερχόμενη ακτίνα

$\theta' < \theta$

$$E\begin{bmatrix}1\\-\cos\theta\\-\sin\theta\\0\end{bmatrix} = p^{\mu}$$

$$E'\begin{bmatrix}1\\-\cos\theta'\\-\sin\theta'\\0\end{bmatrix} = p^{\mu'}$$

$$E'\begin{bmatrix}1\\-\cos\theta'\\-\sin\theta'\\0\end{bmatrix} = \begin{bmatrix}\gamma & -\gamma u & 0 & 0\\-\gamma u & \gamma & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix} E\begin{bmatrix}1\\-\cos\theta\\-\sin\theta\\0\end{bmatrix}$$

$$|\vec{p}'| = E' = \gamma E + \gamma u E\cos\theta = \gamma E(1+u\cos\theta)$$

$$p_x' = -E'\cos\theta' = -\gamma u E - \gamma E\cos\theta = -\gamma E(u+\cos\theta)$$

$$p_y' = -E'\sin\theta' = -E\sin\theta$$

$$p_z' = 0 = 0$$

$$\sin\theta' = \frac{\sin\theta}{\gamma(1+u\cos\theta)} = p_y'/E'$$

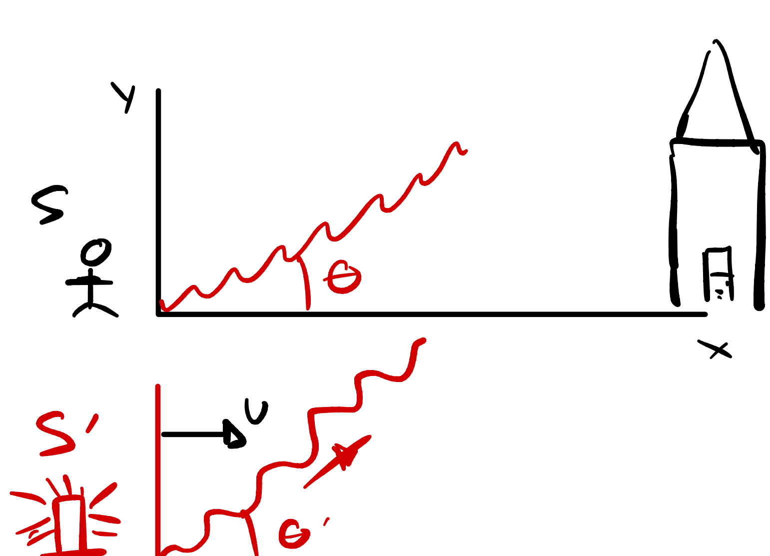
$$\tan\theta' = \frac{\sin\theta}{\gamma(u+\cos\theta)}$$

$$\cos\theta' = \frac{u+\cos\theta}{1+u\cos\theta}$$

$$\boxed{\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}} \quad \checkmark$$

$$\tan(\theta'/2) = \underbrace{\sqrt{\frac{1-u}{1+u}}}_{>1}\tan\left(\frac{\theta}{2}\right)$$

$\tan\theta/2$ είναι γνησίως αύξουσα $0 \leq \theta \leq \pi/2$



για εξερχόμενη ακτίνα

$\theta' > \theta$

$$E\begin{bmatrix}1\\+\cos\theta\\+\sin\theta\\0\end{bmatrix} = p^{\mu}$$

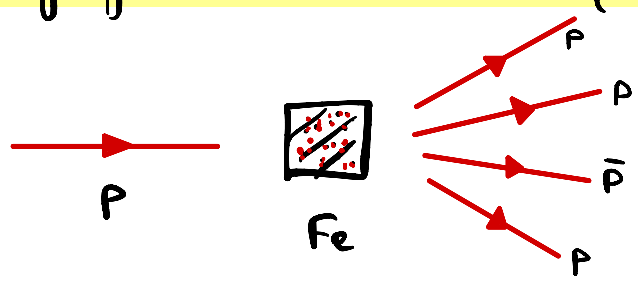
$$E'\begin{bmatrix}1\\+\cos\theta'\\+\sin\theta'\\0\end{bmatrix} = p^{\mu'}$$

$$E\begin{bmatrix}1\\\cos\theta\\\sin\theta\\0\end{bmatrix} = \begin{bmatrix}\gamma & \gamma u & 0 & 0\\\gamma u & \gamma & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix} E'\begin{bmatrix}1\\+\cos\theta'\\+\sin\theta'\\0\end{bmatrix}$$

Ενεργεια κατωφλιου

παραγωγη ενος αντι-πρωτονιου

S



$$\underset{1}{p}\,\underset{2}{p} \to \underset{3}{p}\,\underset{4}{p}\,\underset{5}{\bar{p}}\,\underset{6}{p}$$

$(+\,-\,-\,-)$

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu + p_5^\mu + p_6^\mu$$

$$\left(\begin{pmatrix} E_1 \\ \vec{p}_1 \end{pmatrix} + \begin{pmatrix} m \\ \vec{0} \end{pmatrix} \right)^2 = \left(p_3^\mu + p_4^\mu + p_5^\mu + p_6^\mu \right)^2$$

$$m^2 + m^2 + 2(E_1^{min} m - \vec{p}_1 \vec{0}) = \left(\begin{pmatrix} m \\ 0 \end{pmatrix} + \begin{pmatrix} m \\ 0 \end{pmatrix} + \begin{pmatrix} m \\ 0 \end{pmatrix} + \begin{pmatrix} m \\ 0 \end{pmatrix} \right)^2_{KO}$$

$$E_1^{min} = ? \qquad = (4m)^2$$

$$2m^2 + 2E_1^{min} = 16m^2$$

$$E_1^{min} = 7m \simeq 7\,\text{GeV}$$

$pp \to pp\,\pi^0$

$\gamma e^- \to e^- e^+ e^-$

Εαν εχουμε

$(E, \vec{p})$ $\quad$ $(E, -\vec{p})$

$1 \qquad 2$

$pp \to X$

$$2E^{min} = M_X$$

πχ για $X = pp\bar{p}p$ $\quad E^{min} = 2\,\text{GeV}$

$$p_1^\mu + p_2^\mu = (2E, \vec{p} - \vec{p}) = (\sqrt{s}, \vec{0})$$

Εξισώσεις Maxwell   Διάλεξη 11η   11.01.2022

(1) $\bar\nabla \cdot \bar E = \rho / \varepsilon_0$

(2) $\bar\nabla \cdot \bar B = 0$

(3) $\bar\nabla \times \bar E + \frac{\partial \bar B}{\partial t} = 0$

(4) $\bar\nabla \times \bar B - \mu_0 \varepsilon_0 \frac{\partial \bar E}{\partial t} = \mu_0 \bar J$

αναλλοίωτες σε μετ. Lorentz

* εξίσωση συνέχειας

* πρόβλεψη Η/Μ κυμάτων στο κενό

$\bar\nabla = (\partial_x, \partial_y, \partial_z)$

$\vec F_L = q(\bar E + \bar v \times \bar B)$ ← δύναμη Lorentz

$\bar\nabla(4) \to \bar\nabla\left(\bar\nabla\times\bar B - \mu_0\varepsilon_0 \frac{\partial \bar E}{\partial t}\right) = \bar\nabla(\mu_0 \bar J)$

$\bar\nabla(\bar\nabla\times\bar B) = 0$ (1)

$-\mu_0\varepsilon_0 \frac{\partial \bar\nabla \bar E}{\partial t} = \mu_0 \bar\nabla \bar J$

$-\mu_0\varepsilon_0 \frac{\partial(\rho/\varepsilon_0)}{\partial t} = \mu_0 \bar\nabla \bar J$

* $\boxed{\frac{\partial \rho}{\partial t} + \bar\nabla \bar J = 0}$ Εξ. συνέχειας
τοπική διατήρηση του ηλ. φορτίου

$\bar\nabla \times (4)$ $\bar\nabla\times\left(\bar\nabla\times\bar B - \mu_0\varepsilon_0\frac{\partial \bar E}{\partial t}\right) = \mu_0 \bar J = 0$ στο κενό $\rho = 0$ $\bar J = \bar 0$

$\bar\nabla\times(\bar\nabla\times\bar B) = -\nabla^2\bar B - \mu_0\varepsilon_0 \frac{\partial(\bar\nabla\times\bar E)}{\partial t} = -\nabla^2 \bar B - \mu_0\varepsilon_0 \frac{\partial}{\partial t}\left(-\frac{\partial \bar B}{\partial t}\right) = 0$ (3)

$\bar A \times (\bar B \times \bar C) = \bar B(\bar A \bar C) - \bar C(\bar A \bar B)$ ταυτότητα

$\bar\nabla \times (\bar\nabla \times \bar B) = \bar\nabla(\bar\nabla \bar B) - \nabla^2 \bar B$

$\bar\nabla \bar B = 0$

*
$$\boxed{\begin{aligned} \nabla^2 \bar B - \frac{1}{v^2}\frac{\partial^2 \bar B}{\partial t^2} &= 0 \\ \nabla^2 \bar E - \frac{1}{v^2}\frac{\partial^2 \bar E}{\partial t^2} &= 0 \end{aligned}}$$

ομοίως $\nabla \times (3)$

ορίζοντας ως $v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \overset{SI}{\simeq} 3\cdot 10^8\, m/s$

$c = 1$ σύμβαση

$\square \bar B = 0$

$\square \bar E = 0$

Δυναμη Minkowski & τανυστης Η/Μ πεδιου

$$K^\mu = \frac{dP^\mu}{d\tau} = \frac{d}{d\tau}(m U^\mu) = m a^\mu = q F^{\mu\nu} U_\nu$$

όταν $dm/d\tau = 0$

τανυστης δυναμης
τανυστης Η/Μ πεδιου

προδιαγραφες για K^μ

$K^\mu \sim q$

$K^\mu \sim \vec{E}, \vec{B}$

$K^\mu \sim U^\mu$

$$[\Lambda^{\mu'}{}_\mu] = \begin{bmatrix} \gamma & -\gamma u & 0 & 0 \\ -\gamma u & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

αναλλοιωτος τανυστης 1ης ταξεως

$dx^\mu = (dt, d\vec{x})$

μετ. σαν 3-ανυσμα

$dx^{\mu'} = \Lambda^{\mu'}{}_\mu dx^\mu$

προδιαγραφες για $F^{\mu\nu}$

- $F^{\mu'\nu'} = \Lambda^{\mu'}{}_\mu \Lambda^{\nu'}{}_\nu F^{\mu\nu}$ (ισχυει για καθε τανυστη 2ας ταξεως)

  $(\Delta x^{\mu'} = \Lambda^{\mu'}{}_\mu \Delta x^\mu)$

  1ης ταξεως

- $K^\mu U_\mu = \frac{d}{d\tau}(m U^\mu) U_\mu = \frac{1}{2}\frac{d}{d\tau}(m \underbrace{U^\mu U_\mu}_{-1}) = 0$

$U^\mu \cdot U_\mu = -1$

βουβοι δεικτες

$K^\mu \cdot U_\mu = (q F^{\mu\nu} U_\nu) U_\mu$ πρεπει $= 0$ (1)

$= q F^{\nu\mu} U_\mu U_\nu = 0$ (2)

$U_\mu U_\nu = U_\nu U_\mu$

$(1)+(2)$ $q(F^{\mu\nu} + F^{\nu\mu}) U_\mu U_\nu = 0$

$F^{\mu\nu} = -F^{\nu\mu}$ ← αντισυμμετρικος

$F^{\mu\mu} = 0$

$$\begin{array}{c|cccc} & \nu=0 & \nu=1 & \nu=2 & \nu=3 \\ \hline \mu=0 & & & & \\ \mu=1 & & & & \\ \mu=2 & & & & \\ \mu=3 & & & & \end{array}$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & F^{01} & F^{02} & F^{03} \\ -F^{01} & 0 & F^{12} & F^{13} \\ -F^{02} & -F^{12} & 0 & F^{23} \\ -F^{03} & -F^{13} & -F^{23} & 0 \end{bmatrix}$$

$\vec{E}$ polar

axial $\vec{B}$

ψευδοδιανυσμα

← γενικοτερη δυνατη μορφη αντισυμμετρικου τανυστη 2ας ταξεως

διοτι
$-F^{01} = F^{10}$
$-F^{02} = F^{20}$
$-F^{03} = F^{30}$

6 ανεξαρτητες συνιστωσες (βαθμοι ελευθεριας)

$$F^{\mu\nu} \equiv \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}$$

$E_i \equiv F^{0i}$

$B_i \equiv \frac{1}{2}\varepsilon_{ijk} F^{jk}$

Levi-Civita

πληρες αντισυμμετρικο συμβολο στις 3 διαστασεις

$\varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = +1$

$\varepsilon_{213} = \varepsilon_{132} = \varepsilon_{321} = -1$

$\varepsilon_{ijk} = 0$ για οποιοδηποτε αλλο συνδυασμο των ijk

$i,j = \{1,2,3\}$

$E_1 = E_x = F^{01}$ $\quad E_2 = E_y = F^{02}$

$$B_1 = B_x = \frac{1}{2}\varepsilon_{1jk}F^{jk} = \frac{1}{2}(\varepsilon_{123}F^{23} + \varepsilon_{132}F^{32}) = \frac{1}{2}\left[\overset{+1}{\varepsilon_{123}}F^{23} + (\overset{-1}{-\varepsilon_{123}})(-F^{23})\right] = F^{23} = B_x$$

ασκηση

$B_2 = \frac{1}{2}\varepsilon_{2jk}F^{jk} = \dots = F^{31}$

$$T^{\mu'\nu'\kappa'} = (T'^{\mu\nu\kappa}) = \Lambda^{\mu'}{}_{\mu}\Lambda^{\nu'}{}_{\nu}\Lambda^{\kappa'}{}_{\kappa}T^{\mu\nu\kappa}$$ ανταλλοιωτός τανυστής $3^{ης}$ τάξεως

$$T^{\mu'\nu'}{}_{\kappa'} = \Lambda^{\mu'}{}_{\mu}\Lambda^{\nu'}{}_{\nu}\Lambda_{\kappa'}{}^{\kappa}T^{\mu\nu}{}_{\kappa}$$ μεικτός τανυστής $3^{ης}$ τάξεως

μετ. E & B πεδίου ($F^{\mu\nu}$)

$$\begin{matrix}\bar{E} \\ \bar{B}\end{matrix} \xrightarrow{?} \begin{matrix}\bar{E}' \\ \bar{B}'\end{matrix}$$

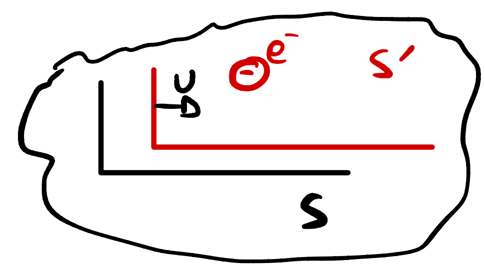
$$\boxed{\vec{u} \to \vec{u}'} \longrightarrow \boxed{U^{\mu} \to U^{\mu'}}$$

$$(\gamma, \gamma\vec{u}) \qquad (\gamma', \gamma'\vec{u}')$$

$$F^{\mu'\nu'} = \begin{bmatrix} 0 & E_x' & E_y' & E_z' \\ -E_x' & 0 & B_z' & -B_y' \\ -E_y' & -B_z' & 0 & B_x' \\ -E_z' & B_y' & -B_x' & 0 \end{bmatrix}$$

$$\begin{array}{cc} & \nu = \;\; 0 \quad\;\; 1 \quad\;\; 2 \quad\;\; 3 \\ F^{\mu\nu} = & \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix} \end{array} \begin{array}{l} 0 = \mu \\ 1 \\ 2 \\ 3 \end{array}$$

Τυποποιημένος μετ. Lorentz $\quad \vec{u} = u\hat{x}$



$$\Lambda^{\mu'}{}_{\mu} = \begin{bmatrix} \gamma & -\gamma u & 0 & 0 \\ -\gamma u & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \Lambda = [\Lambda^{\mu'}{}_{\mu}] = [\Lambda^{\nu'}{}_{\nu}]$$

$$F^{0'0'} = \Lambda^{0'}{}_{\mu}\Lambda^{0'}{}_{\nu}F^{\mu\nu} \qquad \underline{F^{0'1'} = \Lambda^{0'}{}_{\mu}\Lambda^{1'}{}_{\nu}F^{\mu\nu}} = E_x'$$

$$\begin{aligned} F^{0'1'} = {} & \Lambda^{0'}{}_{0}\Lambda^{1'}{}_{0}F^{00} + \Lambda^{0'}{}_{0}\Lambda^{1'}{}_{1}F^{01} + \Lambda^{0'}{}_{0}\Lambda^{1'}{}_{2}F^{02} + \Lambda^{0'}{}_{0}\Lambda^{1'}{}_{3}F^{03} \\ & \Lambda^{0'}{}_{1}\Lambda^{1'}{}_{0}F^{10} + \Lambda^{0'}{}_{1}\Lambda^{1'}{}_{1}F^{11} + \Lambda^{0'}{}_{1}\Lambda^{1'}{}_{2}F^{12} + \Lambda^{0'}{}_{1}\Lambda^{1'}{}_{3}F^{13} \\ & \Lambda^{0'}{}_{2}\Lambda^{1'}{}_{0}F^{20} + \Lambda^{0'}{}_{2}\Lambda^{1'}{}_{1}F^{21} + \Lambda^{0'}{}_{2}\Lambda^{1'}{}_{2}F^{22} + \Lambda^{0'}{}_{2}\Lambda^{1'}{}_{3}F^{23} \\ & \Lambda^{0'}{}_{3}\Lambda^{1'}{}_{0}F^{30} + \Lambda^{0'}{}_{3}\Lambda^{1'}{}_{1}F^{31} + \Lambda^{0'}{}_{3}\Lambda^{1'}{}_{2}F^{32} + \Lambda^{0'}{}_{3}\Lambda^{1'}{}_{3}F^{33} \end{aligned}$$

μετ. E & B πεδίου ($F^{\mu\nu}$)

$$
\begin{aligned}
F^{0'1'} = {} & \Lambda^{0'}{}_{0}\,\Lambda^{1'}{}_{0}\,\cancel{F^{00}} + \overset{\gamma}{\Lambda^{0'}{}_{0}}\,\overset{\gamma}{\Lambda^{1'}{}_{1}}\,\overset{E_x}{F^{01}} + \Lambda^{0'}{}_{0}\,\cancel{\Lambda^{1'}{}_{2}}\,F^{02} + \Lambda^{0'}{}_{0}\,\cancel{\Lambda^{1'}{}_{3}}\,F^{03} \\
& \overset{-\gamma u}{\Lambda^{0'}{}_{1}}\,\overset{-\gamma u}{\Lambda^{1'}{}_{0}}\,\overset{(-E_x)}{F^{10}} + \Lambda^{0'}{}_{1}\,\Lambda^{1'}{}_{1}\,\cancel{F^{11}} + \Lambda^{0'}{}_{1}\,\cancel{\Lambda^{1'}{}_{2}}\,F^{12} + \Lambda^{0'}{}_{1}\,\cancel{\Lambda^{1'}{}_{3}}\,F^{13} \\
& \cancel{\Lambda^{0'}{}_{2}}\,\Lambda^{1'}{}_{0}\,F^{20} + \cancel{\Lambda^{0'}{}_{2}}\,\Lambda^{1'}{}_{1}\,F^{21} + \Lambda^{0'}{}_{2}\,\Lambda^{1'}{}_{2}\,\cancel{F^{22}} + \Lambda^{0'}{}_{2}\,\cancel{\Lambda^{1'}{}_{3}}\,F^{23} \\
& \cancel{\Lambda^{0'}{}_{3}}\,\Lambda^{1'}{}_{0}\,F^{30} + \cancel{\Lambda^{0'}{}_{3}}\,\Lambda^{1'}{}_{1}\,F^{31} + \Lambda^{0'}{}_{3}\,\cancel{\Lambda^{1'}{}_{2}}\,F^{32} + \Lambda^{0'}{}_{3}\,\Lambda^{1'}{}_{3}\,\cancel{F^{33}} \\
= {} & \gamma^2 E_x - (\gamma u)^2 E_x \\
= {} & E_x
\end{aligned}
$$

$$\gamma^2(1-u^2) = 1$$

$$
\Lambda^{\mu'}{}_{\mu} = \begin{bmatrix} \gamma & -\gamma u & 0 & 0 \\ -\gamma u & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
F^{\mu\nu}:\quad
\begin{array}{c|c}
 & \nu = 0 \quad 1 \quad 2 \quad 3 \\
\hline
\begin{array}{c} \mu = 0 \\ 1 \\ 2 \\ 3 \end{array} &
\begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}
\end{array}
$$

$$F^{\mu'\nu'} = \Lambda^{\mu'}{}_{\mu}\,\Lambda^{\nu'}{}_{\nu}\,F^{\mu\nu} \quad \text{"συνιστώσες"} \;\longrightarrow\; \overset{F'}{[F^{\mu'\nu'}]} = \overset{\Lambda}{[\Lambda^{\mu'}{}_{\mu}]}\,\overset{F}{[F^{\mu\nu}]}\,\overset{\Lambda^T}{[\Lambda^{\nu'}{}_{\nu}]^T} \quad \text{"πίνακες"}$$

$$C_{ij} = \sum_k A_{ik} B_{kj} \;\longrightarrow\; [C] = [A]\cdot[B]$$

$$D_{ij} = \sum_k B_{jk} A_{ik} \;\longrightarrow\; [D] = [A]\,[B]^T$$

$$
\overset{F'}{[F^{\mu'\nu'}]} = \overset{\Lambda}{\begin{bmatrix} \gamma & -\gamma u & 0 & 0 \\ -\gamma u & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}
\overset{F}{\begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}}
\overset{\Lambda'}{\begin{bmatrix} \gamma & -\gamma u & 0 & 0 \\ -\gamma u & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}
$$

$$
\begin{bmatrix} \gamma u E_x & \gamma E_x & \gamma E_y - \gamma u B_z & \gamma E_z + \gamma u B_y \\ -\gamma E_y & -\gamma u E_x & -\gamma u E_y + \gamma B_z & -\gamma u E_z - \gamma B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}
\begin{bmatrix} \gamma & -\gamma u & 0 & 0 \\ -\gamma u & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$\begin{bmatrix} \gamma u E_x & \gamma E_x & \gamma E_y - \gamma u B_z & \gamma E_z + \gamma u B_y \\ -\gamma E_y & -\gamma u E_x & -\gamma u E_y + \gamma B_z & -\gamma u E_z - \gamma B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix} \begin{bmatrix} \gamma & -\gamma u & 0 & 0 \\ -\gamma u & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{matrix} \gamma^2 u E_x - \gamma^2 u E_x & -\gamma^2 u^2 E_x + \gamma^2 E_x & \gamma E_y - \gamma u B_z & \gamma E_z + \gamma u B_y \\ -\gamma^2 E_x + \gamma^2 u^2 E_x & \gamma^2 u E_x - \gamma^2 u E_x & -\gamma u E_y + \gamma B_z & -\gamma u E_z - \gamma B_y \\ -\gamma E_y + \gamma u B_z & \gamma u E_y - \gamma B_z & 0 & B_x \\ -\gamma E_z - \gamma u B_y & +\gamma u E_z + \gamma B_y & -B_x & 0 \end{matrix}$$

$$\gamma^2(1-u^2) = 1$$

$$\begin{bmatrix} 0 & E_x & \gamma(E_y - uB_z) & \gamma(E_z + uB_y) \\ -E_x & 0 & \gamma(B_z - uE_y) & -\gamma(B_y + uE_z) \\ -\gamma(E_y - uB_z) & -\gamma(B_z - uE_y) & 0 & B_x \\ -\gamma(E_z + uB_y) & \gamma(B_y + uE_z) & -B_x & 0 \end{bmatrix}$$

$$F^{\mu'\nu'} = \begin{bmatrix} 0 & E_x' & E_y' & E_z' \\ -E_x' & 0 & B_z' & -B_y' \\ -E_y' & -B_z' & 0 & B_x' \\ -E_z' & B_y' & -B_x' & 0 \end{bmatrix}$$

$$\left.\begin{aligned} E_x' &= E_x \\ E_y' &= \gamma(E_y - uB_z) \\ E_z' &= \gamma(E_z + uB_y) \end{aligned}\right\} \quad \begin{aligned} E_\parallel' &= E_\parallel \\ \bar{E}_\perp' &= \gamma(E_\perp + \bar{u}\times\bar{B}) \\ &= \gamma(E_\perp + \bar{u}\times\bar{B}_\perp) \end{aligned}$$

$$\vec{u} = u\hat{x}$$

$$\bar{u}\times\bar{B} = \begin{vmatrix} i & j & k \\ u & 0 & 0 \\ B_x & B_y & B_z \end{vmatrix} = 0\,\hat{i} + (-uB_z)\hat{j} + (uB_y)\hat{k}$$

$$(u\times B)_i = \sum_j \sum_k \varepsilon_{ijk} u_j B_k$$

$$\boxed{\begin{aligned} \vec{E}_\parallel' &= \vec{E}_\parallel \\ \bar{E}_\perp' &= \gamma(\bar{E}_\perp + \bar{u}\times\bar{B}) \\ &= \gamma(\bar{E}_\perp + \bar{u}\times\bar{B}_\perp) \end{aligned}}$$

$$\boxed{\begin{aligned} \bar{B}_\parallel' &= \bar{B}_\parallel \\ \bar{B}_\perp' &= \gamma(\bar{B}_\perp - \bar{u}\times\bar{E}) \\ \bar{B}_\perp' &= \gamma(\bar{B}_\perp - \bar{u}\times\bar{E}_\perp) \end{aligned}}$$

Δυικος Τανυστης

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\kappa\lambda} F_{\mu\nu}$$

$\uparrow$ 4-διαστατο συμβολο Levi-Civita

$\varepsilon^{\mu\nu\kappa\lambda} = (-1)^{p}\,\varepsilon^{0123}$

$\varepsilon^{0123} = +1 \qquad \varepsilon_{0123} = -1$

p = ομοτιμια μεταθεσεων

4^4 στοιχεια

αν δυο δεικτες ειναι ιδιοι

π.χ. $\varepsilon^{0012} = -\varepsilon^{0012} = 0$

τοτε το στοιχειο του τανυστη ειναι μηδεν

$\boxed{\varepsilon_{\mu\nu\kappa\lambda}\,\varepsilon^{\mu\nu\kappa\lambda} = -24 \quad \text{ασκηση}}$

0123
1230
2301

$$F_{\mu\nu} = \eta_{\mu\kappa}\,\eta_{\nu\lambda}\,F^{\kappa\lambda}$$

E^T

$$[F^{\mu\nu}] = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix} = \begin{bmatrix} 0 & E^T \\ -E & \boxed{B} \end{bmatrix} = F$$

$$[\eta_{\mu\nu}] = \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & I \end{bmatrix} = \eta$$

$$[F_{\mu\nu}] = \eta\, F\, \eta^T = \begin{bmatrix} -1 & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} 0 & E^T \\ -E & B \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & I \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} 0 & E^T \\ +E & B \end{bmatrix} = \begin{bmatrix} 0 & -E^T \\ E & B \end{bmatrix}$$

$$F_{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix}$$

$$\tilde{F}^{\mu\nu} = \begin{bmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z & E_y \\ -B_y & E_z & 0 & -E_x \\ -B_z & -E_y & E_x & 0 \end{bmatrix}$$

$$\tilde{F}^{00} = \frac{1}{2}\varepsilon^{00\kappa\lambda}F_{\kappa\lambda} = 0$$

$$\tilde{F}^{01} = \frac{1}{2}\varepsilon^{01\kappa\lambda}F_{\kappa\lambda} = \frac{1}{2}\left(\varepsilon^{0123}F_{23} + \varepsilon^{0132}F_{32}\right)$$

$$\frac{1}{2}\left(\varepsilon^{0123}F_{23} + (-\varepsilon^{0123})(-F_{23})\right) = F_{23} = B_x$$

$$\tilde{F}^{02} = \frac{1}{2}\varepsilon^{02\kappa\lambda}F_{\kappa\lambda} = \frac{1}{2}\left(\varepsilon^{0213}F_{13} + \varepsilon^{0231}F_{31}\right) = F_{31} = B_y$$

$$\tilde{F}^{03} = \frac{1}{2}\varepsilon^{03\kappa\lambda}F_{\kappa\lambda} = {}_2\left(\varepsilon^{0312}F_{12} + \varepsilon^{0321}F_{21}\right) = F_{12} = B_z$$

$$0312 \;\perp\; 0132 \;\perp\; 0123$$

Μετασχηματισμοί Lorentz & εξ. Maxwell

Εξετάζω τον μετασχηματισμό της $\bar{\nabla}\cdot\bar{B}=0 \to \bar{\nabla}'\bar{B}' = ?$

$\partial_x = \frac{\partial}{\partial x}$ για $\vec{u} = u\hat{x}$

$$\bar{\nabla} = (\partial_x, \partial_y, \partial_z) \to \bar{\nabla}' = \left(\frac{\partial}{\partial x'}, \frac{\partial}{\partial y'}, \frac{\partial}{\partial z'}\right)$$

$$\bar{B} \to \bar{B}' \qquad \bar{\nabla}' = \left(\gamma\frac{\partial}{\partial x} + \gamma u \frac{\partial}{\partial t}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\frac{\partial}{\partial x'} = \frac{\partial}{\partial x}\underbrace{\frac{\partial x}{\partial x'}}_{\gamma} + \frac{\partial}{\partial t}\underbrace{\frac{\partial t}{\partial x'}}_{\gamma u} = \gamma\frac{\partial}{\partial x} + \gamma u\frac{\partial}{\partial t}$$

$$x = x(x', t') = \gamma(x' + u t')$$
$$t = t(x', t') = \gamma(t' + u x')$$
$$y' = y$$
$$z' = z$$

$$\frac{\partial}{\partial y'} = \frac{\partial}{\partial y}$$
$$\frac{\partial}{\partial z'} = \frac{\partial}{\partial z}$$

$$\hat{x}\times\hat{x} = 0$$
$$\hat{x}\times\hat{y} = \hat{z}$$
$$\hat{x}\times\hat{z} = -\hat{y}$$

$$\bar{B}'_{\parallel} = \bar{B}_{\parallel} = B_x\hat{x}$$

$$\bar{B}'_{\perp} = \gamma(\bar{B}_{\perp} - \bar{u}\times\bar{E}) = \gamma\left(B_y\hat{y} + B_z\hat{z} - u\hat{x}\times(E_x\hat{x} + E_y\hat{y} + E_z\hat{z})\right)$$

$$\bar{B}' = \left(B_x, \gamma(B_y + uE_z), \gamma(B_z - uE_y)\right)$$

$$\bar{\nabla}'\bar{B}' = \left(\gamma\frac{\partial}{\partial x} + \gamma u\frac{\partial}{\partial t}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\left(B_x, \gamma(B_y + uE_z), \gamma(B_z - uE_y)\right)$$

$$= \gamma\frac{\partial B_x}{\partial x} + \gamma u\frac{\partial B_x}{\partial t} + \gamma\frac{\partial B_y}{\partial y} + \gamma u\frac{\partial E_z}{\partial y} + \gamma\frac{\partial B_z}{\partial z} - \gamma u\frac{\partial E_y}{\partial z}$$

$$= \gamma\frac{\partial B_x}{\partial x} + \gamma\frac{\partial B_y}{\partial y} + \gamma\frac{\partial B_z}{\partial z} + \gamma u\frac{\partial E_z}{\partial y} + \gamma u\frac{\partial B_x}{\partial t} - \gamma u\frac{\partial E_y}{\partial z}$$

$$= \gamma(\bar{\nabla}\bar{B}) + \gamma u\left(\underbrace{\frac{\partial B_x}{\partial t}}_{\left.\frac{\partial\bar{B}}{\partial t}\right|_x} + \underbrace{(\partial_y E_z - \partial_z E_y)}_{(\bar{\nabla}\times\bar{E})_x}\right)$$

$$= \gamma(\bar{\nabla}\bar{B}) + \gamma u\left[\frac{\partial\bar{B}}{\partial t} + (\bar{\nabla}\times\bar{E})\right]_x = 0$$

$$\bar{\nabla}\bar{B} = 0$$
$$\bar{\nabla}\times\bar{E} + \frac{\partial\bar{B}}{\partial t} = 0$$

$$\begin{cases} \bar{\nabla}\cdot\bar{E} = 0 \\ \bar{\nabla}\cdot\bar{B} = 0 \\ \bar{\nabla}\times\bar{E} + \dfrac{\partial\bar{B}}{\partial t} = 0 \\ \bar{\nabla}\times\bar{B} - \mu_0\varepsilon_0\dfrac{\partial\bar{E}}{\partial t} = 0 \end{cases}$$

$\rho = 0 \qquad \bar{J} = \bar{0}$

$(\mu_0\varepsilon_0)^{-1/2} = c = 1$

Εξισώσεις του Maxwell στο κενό

$\partial_\mu F^{\mu\nu} = 0 \longrightarrow \bar{\nabla}\bar{E} = 0 \ \Big| \ \dfrac{\partial\bar{E}}{\partial t} - \bar{\nabla}\times\bar{B} = 0$

$\partial_\mu \tilde{F}^{\mu\nu} = 0 \longrightarrow \bar{\nabla}\bar{B} = 0 \ \Big| \ \dfrac{\partial\bar{B}}{\partial t} + \bar{\nabla}\times\bar{E} = 0$

$\nu = 0$

$$\partial_\mu F^{\mu 0} = 0$$
$$\partial_t F^{00} + \partial_x F^{10} + \partial_y F^{20} + \partial_z F^{30} = 0$$
$$0 + \partial_x E_x + \partial_y E_y + \partial_z E_z = 0$$
$$\bar{\nabla}\cdot\bar{E} = 0$$

$\nu = 1$

$$\partial_\mu F^{\mu 1} = \partial_t F^{01} + \partial_x F^{11} + \partial_y F^{21} + \partial_z F^{31}$$
$$= \partial_t E_x + 0 + \partial_y(-B_z) + \partial_z B_y$$
$$= \left[\frac{\partial}{\partial t}\bar{E} - (\bar{\nabla}\times\bar{B})\right]_x$$

$$\begin{array}{c} \\ \\ \mu=2 \\ \mu=3 \end{array} \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix} =$$

$\nu = 1$

$\nu = 1$

αναλλοίωτα του Η/Μ πεδίου

- $\frac{1}{2} F^{\mu\nu} F_{\mu\nu} = \frac{1}{2}\left(-\vec{E}\cdot\vec{E} + \vec{B}\cdot\vec{B} - \vec{E}\cdot\vec{E} + \vec{B}\cdot\vec{B}\right) = -E^2 + B^2 \equiv I_1$
- $\frac{1}{4} \tilde{F}^{\mu\nu} F_{\mu\nu} = \frac{1}{4}\left(-\vec{E}\cdot\vec{B} - \vec{E}\cdot\vec{B} - \vec{E}\cdot\vec{B} - \vec{E}\cdot\vec{E}\right) = -\vec{E}\cdot\vec{B} \equiv I_2$
- $\frac{1}{2} F^{\mu}{}_{\nu} F^{\nu}{}_{\mu} = \frac{1}{2} \eta_{\nu\kappa} F^{\mu\kappa} F^{\nu}{}_{\mu} = \frac{1}{2} F^{\mu\kappa} \eta_{\nu\kappa} F^{\nu}{}_{\mu} = \frac{1}{2} F^{\mu\kappa} F_{\kappa\mu} = \frac{1}{2} F^{\mu\nu} F_{\nu\mu} = \frac{1}{2} F^{\mu\nu}(-F_{\mu\nu}) = -I_1$
- $\eta_{\mu\kappa} F^{\mu\kappa} = F^{\mu}{}_{\mu} = tr([F^{\mu\nu}]) = 0+0+0+0$
- $F^{\alpha\beta} F_{\beta\gamma} F^{\gamma}{}_{\alpha} = g(I_1, I_2)$
- $F^{\alpha\beta} F_{\beta\gamma} F^{\gamma\delta} F_{\delta\alpha} = f(I_1, I_2)$

F $\qquad \eta = \begin{bmatrix} -1 & 0 \\ 0 & I \end{bmatrix}$

$$[F^{\mu\nu}] = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix} \qquad [F_{\mu\nu}] = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix} \qquad [\tilde{F}^{\mu\nu}] = \begin{bmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z & E_y \\ -B_y & E_z & 0 & -E_x \\ -B_z & -E_y & E_x & 0 \end{bmatrix}$$

τα συναρτησιακώς ανεξάρτητα αναλλοίωτα είναι το πολύ δύο
(Θεώρημα 6.5.1 σημειώσεις Χριστοδουλάκη & Κορφιάτη)

περίπτωση με $\vec{E}\cdot\vec{B} \neq 0$

υπάρχει S' στο οποίο $\vec{E}' \parallel \vec{B}'$

π.χ. για $-B^2 + E^2 > 0$

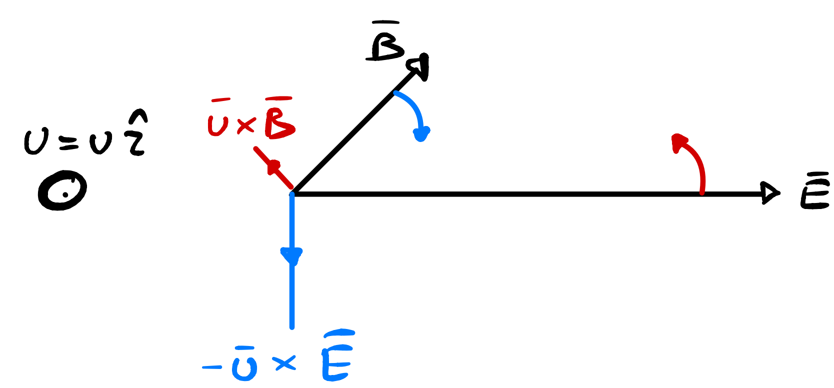
$\vec{B}' = \gamma(\vec{B} - \vec{u}\times\vec{E})$

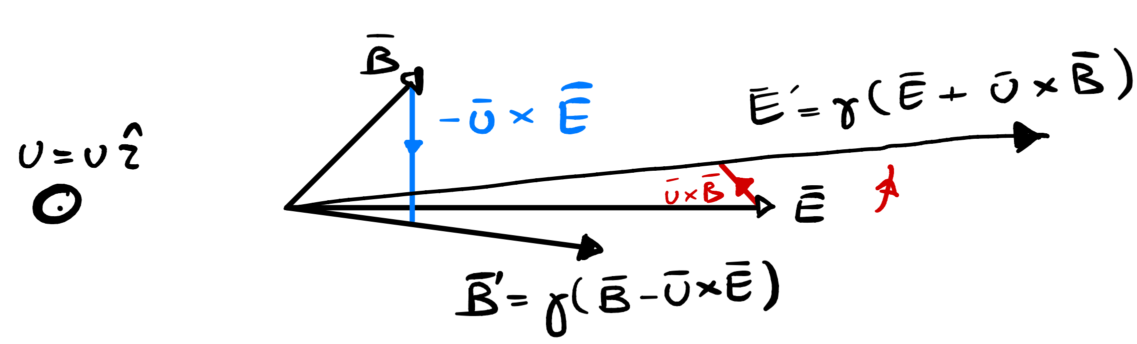
$\vec{E}' = \gamma(\vec{E} + \vec{u}\times\vec{B})$

Έστω ότι $\vec{E}\times\vec{B} / EB\sin\phi = \hat{z}$
και $\vec{u} = u\hat{z}$

$E_\parallel' = E_\parallel = 0 \qquad B_\parallel' = B_\parallel = 0$

$\vec{E} = \vec{E}_T \qquad \vec{B} = \vec{B}_T$





$F_\mu{}^\nu = \eta_{\mu\kappa} F^{\kappa\nu}$ $\qquad F_\mu{}^\mu$

$[F_\mu{}^\nu] = \eta \cdot F$

$$= \begin{bmatrix} -1 & 0 \\ 0 & I \end{bmatrix} \cdot \begin{bmatrix} 0 & E^T \\ -E & B \end{bmatrix} = \begin{bmatrix} 0 & -E^T \\ -E & B \end{bmatrix} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}$$

$\eta \cdot F$ $\qquad AB \neq BA$

$F \cdot \eta$ $\qquad AB - BA = 0$

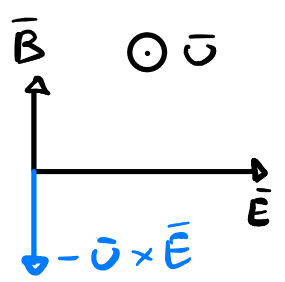
$$F_\mu{}^\mu = tr\left(\begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix} \right) = 0 + 0 + 0 + 0$$

Περιπτωση με $\bar{E}\cdot\bar{B}=0$

$E^2 \neq B^2$

$E^2 > B^2$

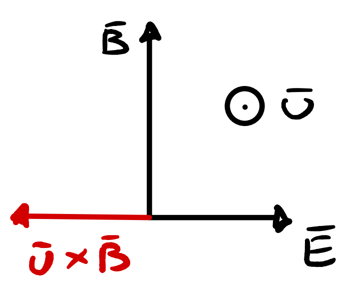
υπαρχει S'
στο οποιο
$\bar{B}'=0$



$$\bar{B}'=\gamma(\bar{B}-\bar{\upsilon}\times\bar{E})$$
$$=0$$

$E^2 < B^2$

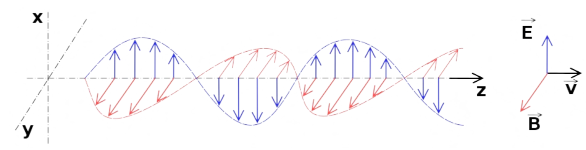
υπαρχει S'
στο οποιο
$\bar{E}'=0$



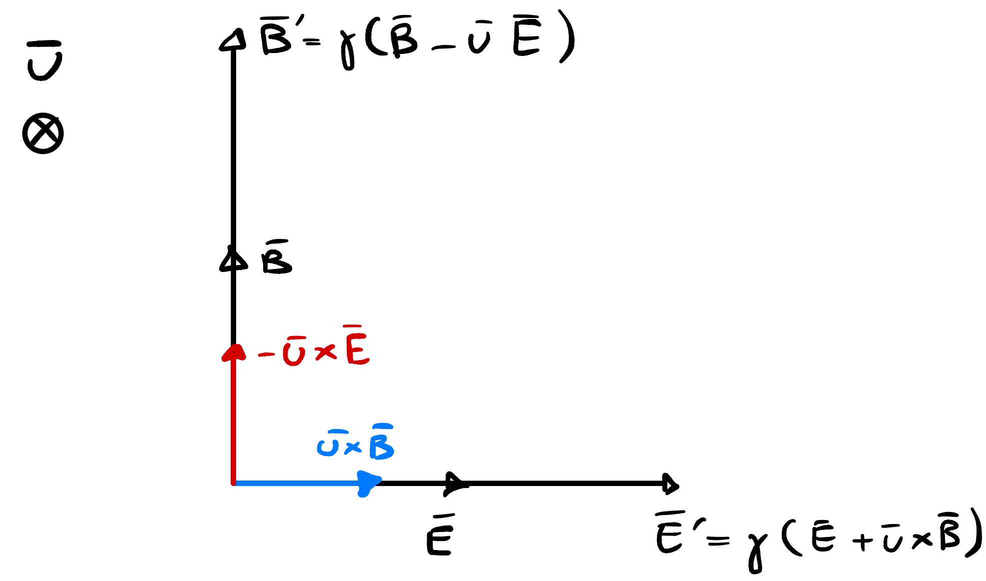
$$\bar{E}'=\gamma(\bar{E}+\bar{\upsilon}\times\bar{B})$$
$$=0$$

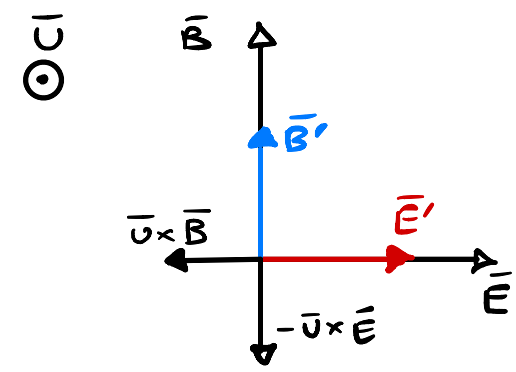
$E^2 = B^2$

ΗΜ κυματα



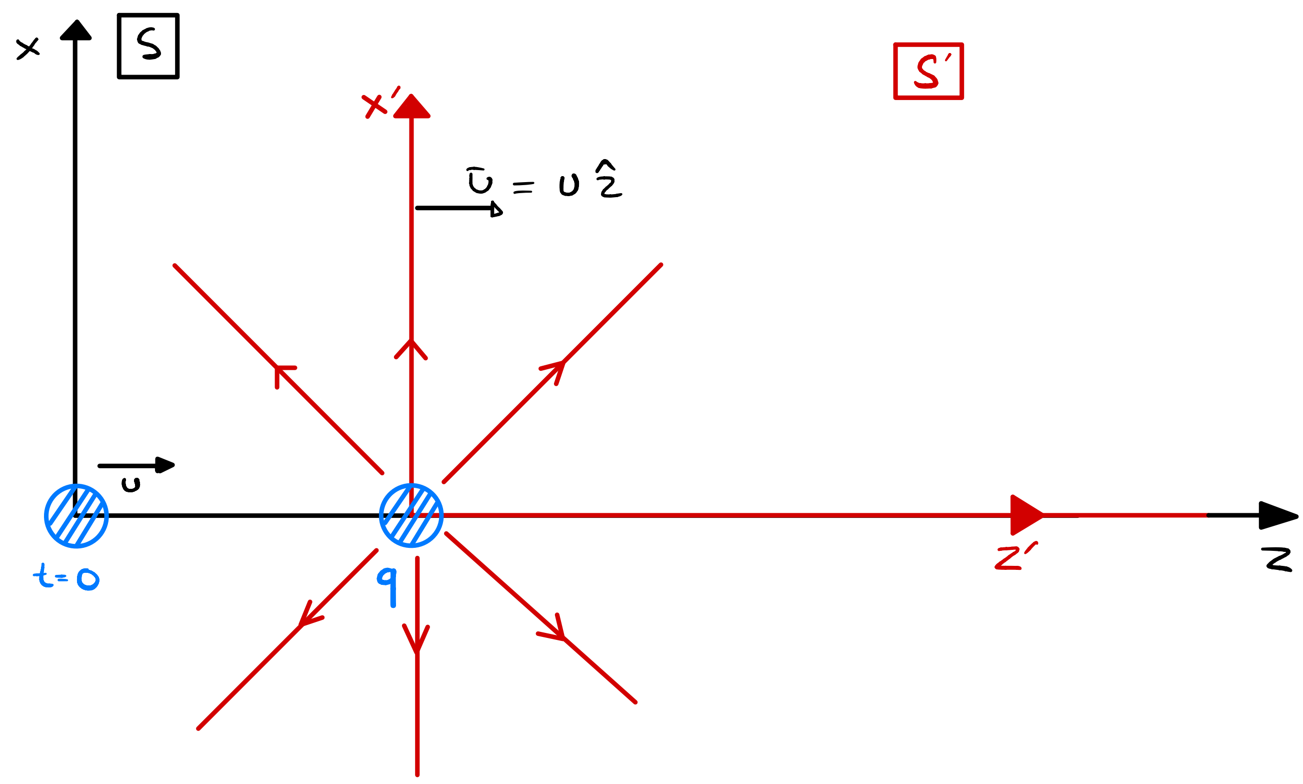
σταθ. επιπεδο πολωσης





Η/Μ πεδίο κινούμενου φορτίου

διάλεξη 14η 19.01.2022



$\boxed{S'}$

$\vec{E}' = \frac{q}{4\pi\varepsilon_0}\frac{\hat{r}'}{(r')^2}$

$\frac{\hat{r}'}{r'^2} = \frac{\vec{r}'}{r'^3} = \frac{(x', y', z')}{((x')^2+(y')^2+(z')^2)^{3/2}}$

$\bar{E}_{\parallel} = \bar{E}_{\parallel}'$

$\bar{E}_{\perp} = \gamma(\bar{E}_{\perp}' - \bar{υ}\times\bar{B}')$

$\bar{B}_{\parallel} = \bar{B}_{\parallel}'$

$\bar{B}_{\perp} = \gamma(\cancel{\bar{B}_{\perp}}^{\,0} + \underbrace{\bar{υ}\times\bar{E}})$

$\boxed{S}$

$E_x = \gamma(E_x' - \bar{υ}\times\cancel{B_T'}^{\,0}) = \gamma\frac{A\,x'}{((x')^2+(y')^2+(z')^2)^{3/2}}$ $\qquad A \equiv \frac{q}{4\pi\varepsilon_0}$

$E_y = \gamma(E_y' - \bar{υ}\times\cancel{B_T'}^{\,0}) = \gamma\frac{A\,y'}{((x')^2+(y')^2+(z')^2)^{3/2}}$

$E_z = E_z' = A\frac{z'}{((x')^2+(y')^2+(z')^2)^{3/2}}$

$z' = \gamma(z - vt)$
$x' = x$
$y' = y$

$$E_x\Big|_{t=0} = \gamma \frac{A\,x}{((x)^2+(y)^2+(\gamma z)^2)^{3/2}}$$

$$E_y\Big|_{t=0} = \gamma \frac{A\,y}{((x)^2+(y)^2+(\gamma z)^2)^{3/2}}$$

$$E_z\Big|_{t=0} = \gamma \frac{A\,z}{((x)^2+(y)^2+(\gamma z)^2)^{3/2}}$$

$$\vec{E} = \gamma \frac{A}{((x)^2+(y)^2+(\gamma z)^2)^{3/2}} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

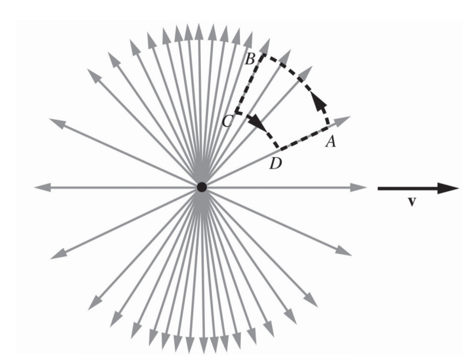
$x = r\sin\theta\cos\phi$
$y = r\sin\theta\sin\phi$
$z = r\cos\theta$

$$\boxed{\vec{E} = \frac{1}{\gamma^2(1 - v^2\sin^2\theta)^{3/2}} \frac{q}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2}}$$

$$((x)^2+(y)^2+(\gamma z)^2)^{3/2} = \left[r^2\sin^2\theta + \gamma^2 r^2\cos^2\theta\right]^{3/2} = r^3\left(\sin^2\theta + \gamma^2(1-\sin^2\theta)\right)^{3/2}$$
$$= r^3\left(\gamma^2 - \gamma^2 v^2 \sin^2\theta\right)^{3/2}$$
$$r^3\gamma^3(1 - v^2\sin^2\theta)^{3/2}$$

$\theta = 0$ $\quad \vec{E} = \frac{1}{\gamma^2} A \frac{\hat{r}}{r^2}$

$\theta = \frac{\pi}{2}$ $\quad \vec{E} = A\frac{\hat{r}}{r^2}$



$\bar{B}' = 0$ $\quad \bar{B}'_{\parallel} = 0 = \bar{B}_{\parallel}$
$\bar{B}'_{\perp} = 0 = \gamma(\bar{B}_{\perp} - \bar{v}\times\bar{E}) = 0$

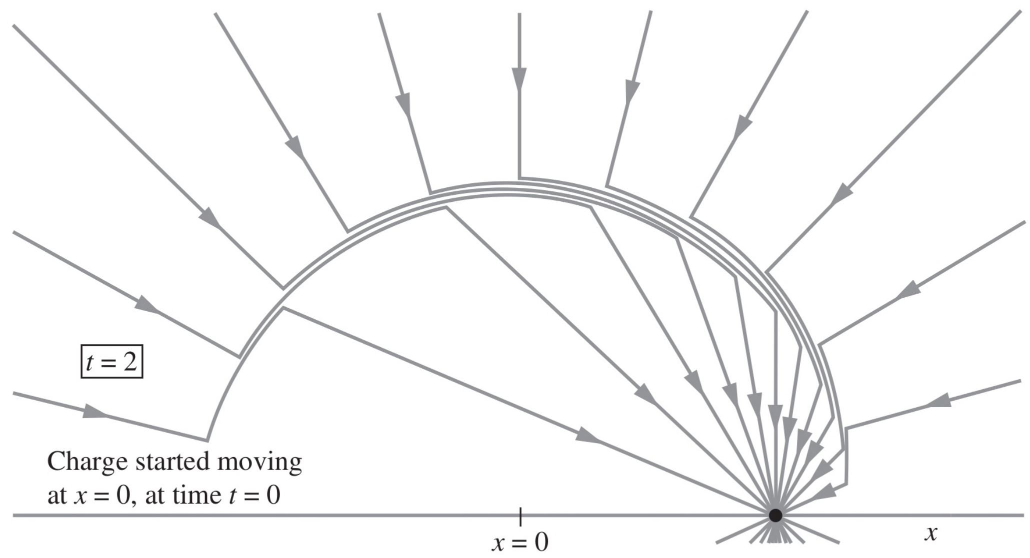
$\bar{B} = \bar{v}\times\bar{E}$ όταν $\bar{B}' = 0$

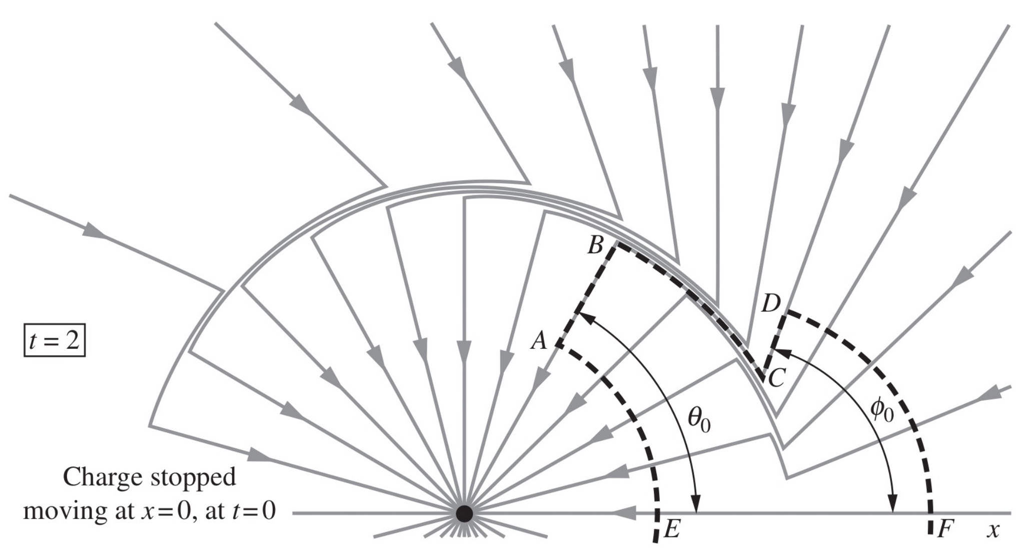
$$\boxed{\bar{B} = \frac{v\sin\theta}{\gamma^2(1 - v^2\sin^2\theta)^{3/2}} \frac{q}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2}\,\hat{\phi}}$$

$v \cdot E \cdot (\hat{z}\times\hat{r})$
$\quad \hookrightarrow \sin\theta\,\hat{\phi}$

Electricity & Magnetism

Purcell M. Edward





Εξίσωση κίνησης φορτίου εντός Η/Μ πεδίου

$$\frac{dp^{\mu}}{d\tau} = \frac{d}{d\tau}\begin{bmatrix} E \\ \vec{p} \end{bmatrix} = \begin{bmatrix} \gamma\, dE/dt \\ \gamma\, d\vec{p}/dt \end{bmatrix} = q\, F^{\mu\nu} \cdot u_{\nu} = q \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix} \begin{bmatrix} -\gamma \\ \gamma u_x \\ \gamma u_y \\ \gamma u_z \end{bmatrix}$$

$$u_{\nu} = \eta_{\mu\nu} u^{\mu}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \gamma \\ \gamma \vec{u} \end{bmatrix}$$

$$= \begin{bmatrix} \gamma(E_x u_x + E_y u_y + E_z u_z) \\ \gamma(E_x + B_z u_y - B_y u_z) \\ \gamma(E_y - B_z u_x + B_x u_z) \\ \gamma(E_z + B_y u_x - B_x u_y) \end{bmatrix} = \begin{bmatrix} \gamma(q\vec{E}\cdot\vec{u}) \\ \gamma q(\vec{E} + \vec{u}\times\vec{B}) \end{bmatrix} = \begin{bmatrix} \gamma(\vec{F}_L \cdot \vec{u}) \\ \gamma \vec{F}_L \end{bmatrix}$$

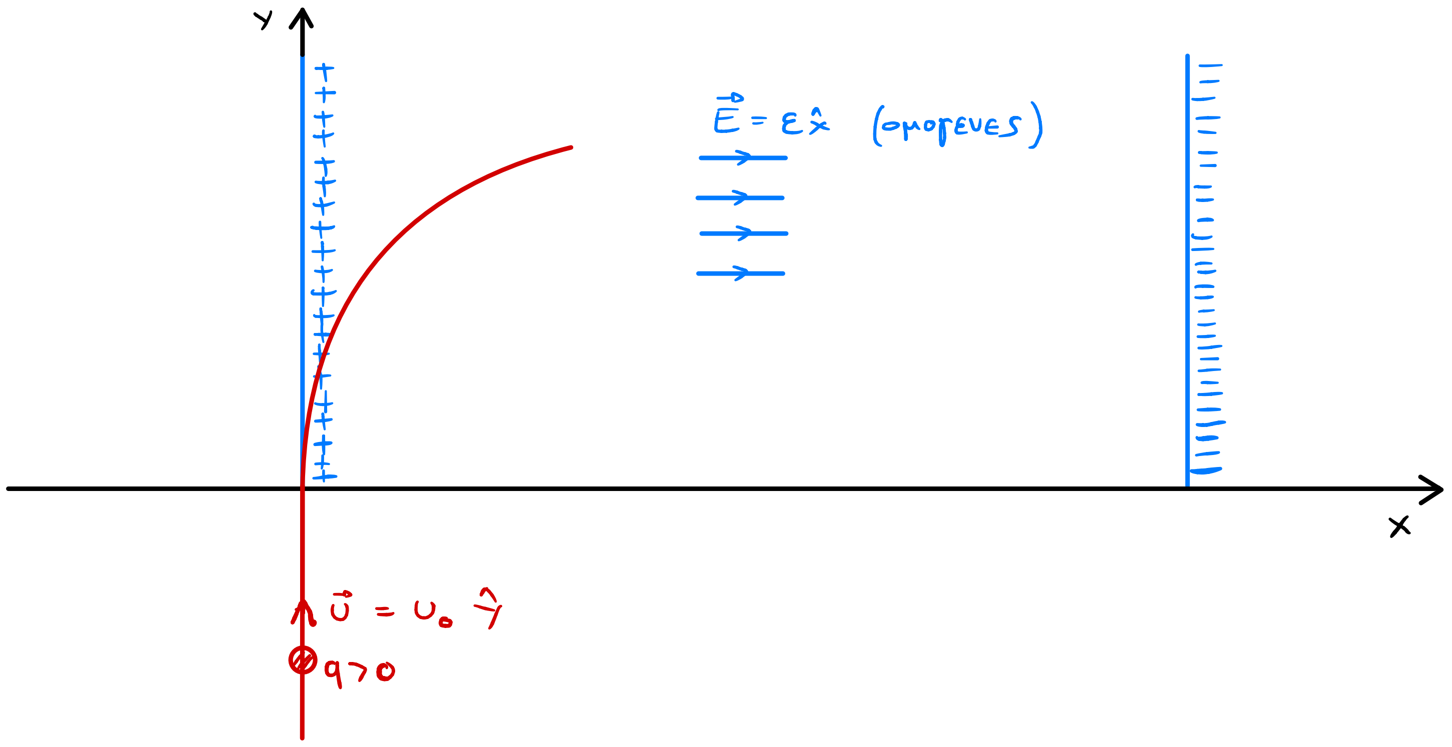
$$(\bar{u} \times \bar{B})_i = \varepsilon_{ijk} u_j B_k$$

$$(\bar{u}\times\bar{B})_x = u_y B_z - u_z B_y$$
$$(\bar{u}\times\bar{B})_y = B_x u_z - B_z u_x$$
$$(\bar{u}\times\bar{B})_z = B_y u_x - B_x u_y$$

$$\vec{u}\cdot\vec{F}_L = \vec{u}\, q(\vec{E} + \vec{u}\times\vec{B}) = q\,\vec{E}\cdot\vec{u}$$

$$\boxed{d\vec{p}/dt = \frac{d(\gamma m \vec{u})}{dt} = \vec{F}_L = q(\vec{E} + \vec{u}\times\vec{B})}$$

Κινηση φορτιου σε σταθερο Ε-πεδιο



$$\vec{F}_L = q(\vec{E} + \vec{u} \times \vec{B}) = q\,\mathcal{E}\,\hat{x} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(\gamma m \vec{u})$$

$x(t)$, $y(t)$ = ?
$u_x(t)$, $u_y(t)$ = ?

A:	$u_y(t)$ = σταθ	46%
B:	$u_y(t)$ = αυξουσα	15%
C:	$u_y(t)$ = φθινουσα	39%

$$\frac{dP^\mu}{d\tau} = \gamma \begin{bmatrix} dE/dt \\ dP_x/dt \\ dP_y/dt \\ dP_z/dt \end{bmatrix} = q \begin{bmatrix} 0 & \mathcal{E} & 0 & 0 \\ -\mathcal{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \gamma \begin{bmatrix} -1 \\ u_x \\ u_y \\ u_z \end{bmatrix}$$

$$dE/dt = q\,\mathcal{E}\,u_x = (q\,\mathcal{E}\hat{x})\cdot(u_x\hat{x}) = \vec{F}\cdot\vec{u}$$

$$\dot{P}_x = q\,\mathcal{E} \rightsquigarrow P_x = q\,\mathcal{E}t + \cancel{\sigma\tau\alpha\theta}^{\,0}$$

$$\dot{P}_y = 0 \rightsquigarrow P_y = \sigma\tau\alpha\theta = \frac{m u_0}{\sqrt{1-u_0^2}} = \frac{m\,u_y(t)}{\sqrt{1-u^2}} = P_{y0}$$

$$\dot{P}_z = 0 \rightsquigarrow P_z = \sigma\tau\alpha\theta = 0$$

$$u_x(t) = \frac{dx}{dt} = \frac{P_x}{E} = \frac{q\,\mathcal{E}t}{\sqrt{m^2 + P_{y0}^2 + P_x^2}} = \frac{q\,\mathcal{E}t}{\sqrt{\underbrace{m^2 + P_{y0}^2} + (q\,\mathcal{E}t)^2}} = \frac{q\,\mathcal{E}t}{\sqrt{E_0^2 + (q\,\mathcal{E}t)^2}}$$

$$= \frac{q\,\mathcal{E}t/E_0}{\sqrt{1 + \left(\frac{q\,\mathcal{E}}{E_0}t\right)^2}}$$

$$E_0^2 \equiv m^2 + P_{y0}^2$$

$$A \equiv \frac{q\,\mathcal{E}}{E_0}$$

$$dx = \frac{1}{A}\int \frac{At}{\sqrt{1 + (At)^2}}\, d(At)$$

$$x(t) = \frac{1}{A}\sqrt{1+(At)^2} + \text{σταθ} \qquad \Big|\ x(0) = 0 \Rightarrow \text{σταθ} = -\frac{1}{A}$$

$$x(t) = \frac{1}{A}\left(\sqrt{1+(At)^2} - 1\right)$$

εχουμε $\quad P_y(t) = P_{y0} = \text{σταθ}$

$$dy/dt = \frac{P_y(t)}{E} = \frac{P_{y0}}{\sqrt{m^2 + P_{y0}^2 + P_x^2}}$$

$$= \frac{P_{y0}}{\sqrt{E_0^2 + (q\mathcal{E}t)^2}} = \frac{P_{y0}/E_0}{\sqrt{1+(At)^2}} = u_y(t)$$

$$dy = \int \frac{P_{y0}/E_0}{\sqrt{1+(At)^2}}\, dt$$

$$\sinh w = At \qquad\qquad \frac{dw}{A}\cosh w = dt$$

$$\cosh^2 w = 1 + \sinh^2 w$$

$$\int dy = P_{y0}/E_0 \int \frac{\cosh w}{\sqrt{1+\sinh^2 w}}\, \frac{dw}{A}$$

$$= \frac{P_{y0}}{AE_0}\int dw = \frac{P_{y0}}{AE_0}\, w + \cancel{\text{σταθ}}^{\,0} \qquad y(0) = 0$$

$$y(t) = \frac{P_{y0}}{AE_0}\sinh^{-1}(At)$$

$$\boxed{\begin{array}{l} \text{arcsinh} \\ \text{arsinh} \\ \sinh^{-1} \end{array}}$$

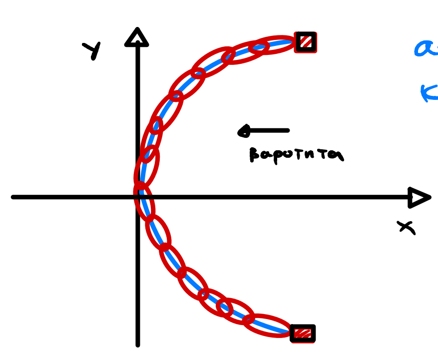
$$x(t) = \frac{E_0}{q\mathcal{E}}\left(\sqrt{1+\left(\frac{q\mathcal{E}t}{E_0}\right)^2} - 1\right) \qquad y(t) = \frac{P_{y0}}{q\mathcal{E}}\sinh^{-1}\left(\frac{q\mathcal{E}t}{E_0}\right)$$

$x(y)$

$$\sinh\left(\frac{q\mathcal{E}\cdot y}{P_{y0}}\right) = \frac{q\mathcal{E}t}{E_0}$$

$$x(y) = \frac{E_0}{q\mathcal{E}}\left(\sqrt{1+\left(\sinh\left(\frac{q\mathcal{E}\cdot y}{P_{y0}}\right)\right)^2} - 1\right)$$

$$x(y) = \frac{E_0}{q\mathcal{E}}\left[\cosh\left(\frac{q\mathcal{E}\cdot y}{P_{y0}}\right) - 1\right]$$

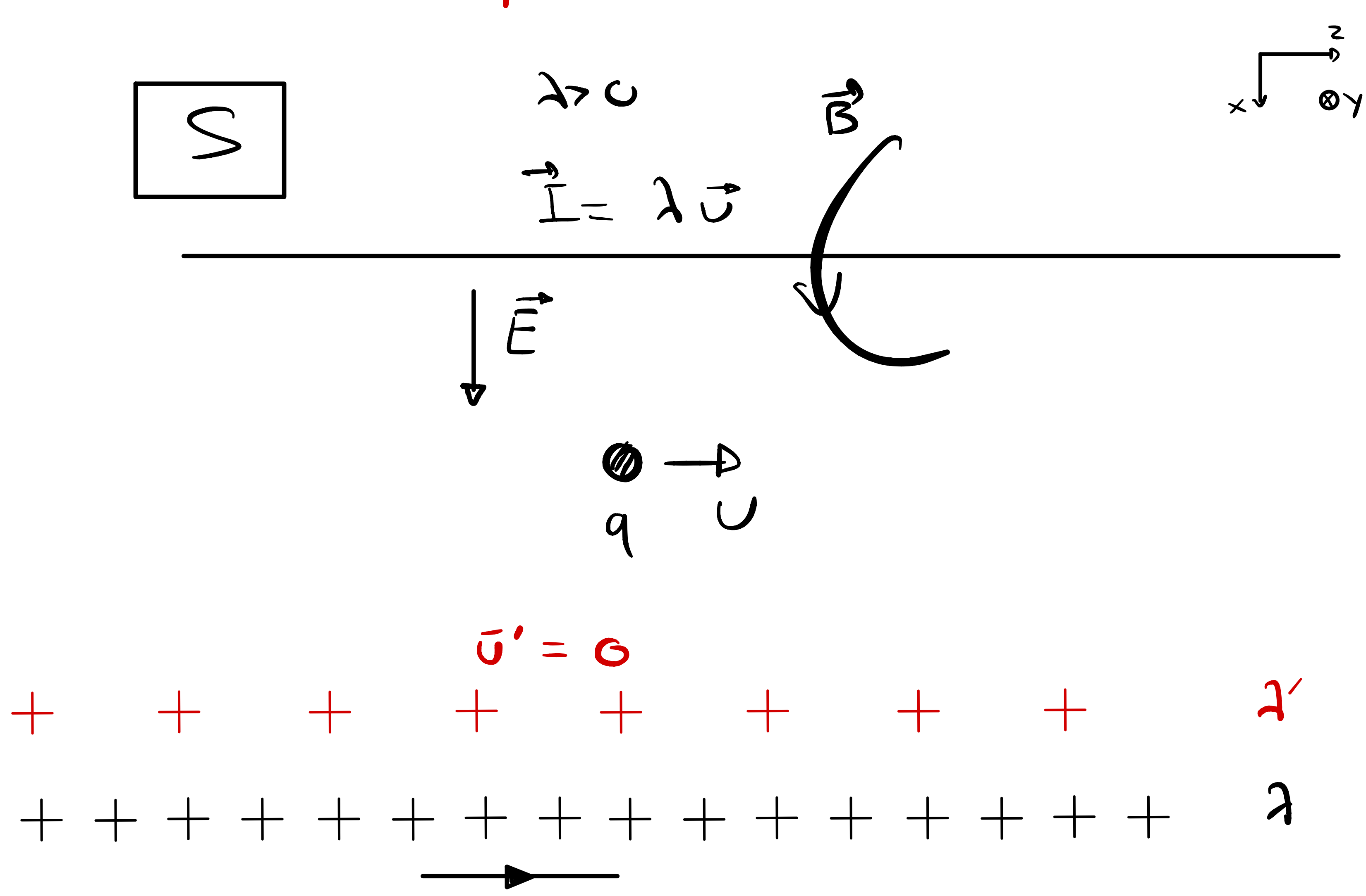


αλυσοειδης καμπυλη

$y(x) \sim \frac{1}{a}\cosh(ax)$

προβλημα εξεταστικη 30.09.2020

↑ λυμενο στο e-class



$$\boxed{\lambda' = \lambda/\gamma}$$