Μαιος 2023

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & 0 \\ 0 & -1 & 4 & -1 & -2 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & -2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} w/k \\ w/k \\ w/k \\ w/k \\ w/k \end{bmatrix}$$

με $w/k = 1$

$$\begin{aligned} 2x_1 - x_2 &= 1 \\ -x_1 + 4x_2 - x_3 &= 1 \\ -x_2 + 4x_3 - x_4 - 2x_5 &= 1 \\ -x_3 + 2x_4 - x_5 &= 1 \\ -2x_3 - x_4 + 3x_5 &= 1 \end{aligned} \Rightarrow$$

$$\begin{aligned} x_1 &= \tfrac{1}{2}(1 + x_2) \\ x_2 &= \tfrac{1}{4}(1 + x_1 + x_3) \\ x_3 &= \tfrac{1}{4}(1 + x_2 + x_4 + 2x_5) \\ x_4 &= \tfrac{1}{2}(1 + x_3 + x_5) \\ x_5 &= \tfrac{1}{3}(1 + 2x_3 + x_4) \end{aligned}$$

```
xi = array (5)
Ji = array (5)
for i in range(5): xi = 0
for n in range( N ):
    for i in range(5):
        Ji = xi

    x1 = 1/2 (1 + J2)
    x2 = 1/4 (1 + J1 + J3)
    x3 = 1/4 (1 + J2 + J4 + 2J5)
    x4 = 1/2 (1 + J3 + J5)
    x5 = 1/3 (1 + 2J3 + J4)

print x1 x2 x3 x4 x5
```

1-st iteration — Jacobi

$x_1 = \frac{1}{2}(1+0) = 1/2$

$x_2 = \frac{1}{4}(1+0) = 1/4$

$x_3 = \frac{1}{4}(1+0+0+0) = 1/4$

$x_4 = \frac{1}{2}(1+0+0) = 1/2$

$x_5 = \frac{1}{3}(1+0+0) = 1/3$

1-st iteration — Gauss-Seidel

$x_1 = \frac{1}{2}(1+0) = 1/2$

$x_2 = \frac{1}{4}(1+\frac{1}{2}+0) = 3/8$

$x_3 = \frac{1}{4}(1+\frac{3}{8}+0+0) = \frac{11}{32}$

$x_4 = \frac{1}{2}(1+\frac{11}{32}+0) = 43/64$

$x_5 = \frac{1}{3}(1+\frac{2\cdot 11}{32}+\frac{43}{64}) =$

$\frac{1}{3}(\frac{64}{64}+\frac{44}{64}+\frac{43}{64}) = \frac{151}{192}$