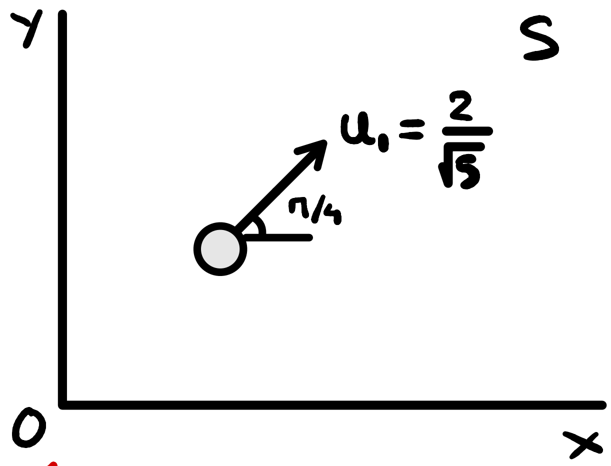
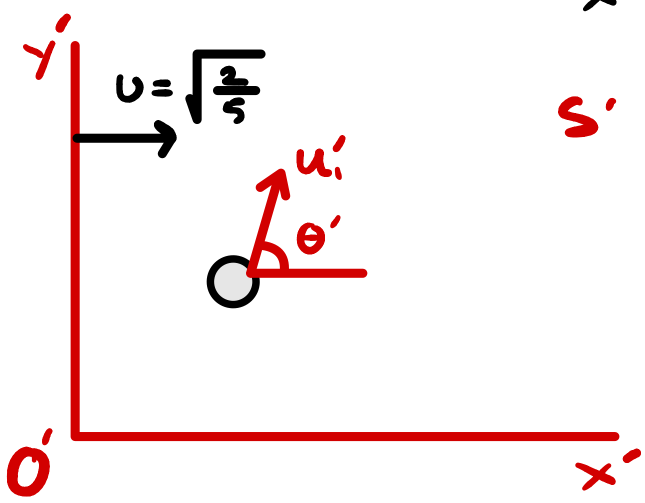
Προβλημα 10.23 (Griffiths)





Βρειτε την εκφωνηση στο αρχειο προβληματων

$$\Lambda\left(\vec{v} = \sqrt{\tfrac{2}{5}}\,\hat{x}\right) \qquad \hat{x}' = \hat{x},\quad \hat{y}' = \hat{y},\quad \hat{z}' = \hat{z}$$

α τροπος

$$\vec{u}_1 = \left(\frac{\sqrt{2}}{\sqrt{5}}, \frac{\sqrt{2}}{\sqrt{5}}, 0\right)$$

$$\gamma_1 = \frac{1}{\sqrt{1-4/5}} =$$

$$u_1^{\mu} = \left(\sqrt{5}, \sqrt{2}, \sqrt{2}, 0\right)$$

$$u_1^{\mu'} = \begin{pmatrix} \sqrt{\frac{5}{3}} & -\sqrt{\frac{2}{3}} & 0 & 0 \\ -\sqrt{\frac{2}{3}} & \sqrt{\frac{5}{3}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} \sqrt{5} \\ \sqrt{2} \\ \sqrt{2} \\ 0 \end{bmatrix}$$

$$\gamma = \frac{1}{\sqrt{1-2/5}} = \sqrt{\frac{5}{3}}$$

$$\gamma v = \sqrt{\frac{5}{3}}\cdot\sqrt{\frac{2}{5}} = \sqrt{\frac{2}{3}}$$

$$= \begin{bmatrix} 5/\sqrt{3} - \frac{2}{\sqrt{3}} \\ -\frac{\sqrt{10}}{\sqrt{3}} + \frac{\sqrt{10}}{\sqrt{3}} \\ \sqrt{2} \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{3} \\ 0 \\ \sqrt{2} \\ 0 \end{bmatrix} = \gamma_1' \begin{bmatrix} 1 \\ u_{1x}' \\ u_{1y}' \\ u_{1z}' \end{bmatrix}$$

$$u_1^{\mu'}\cdot u_{1\mu}' = -1$$

$$-3+2 = -1$$

✓ OK

$$\gamma_1' = \sqrt{3}$$

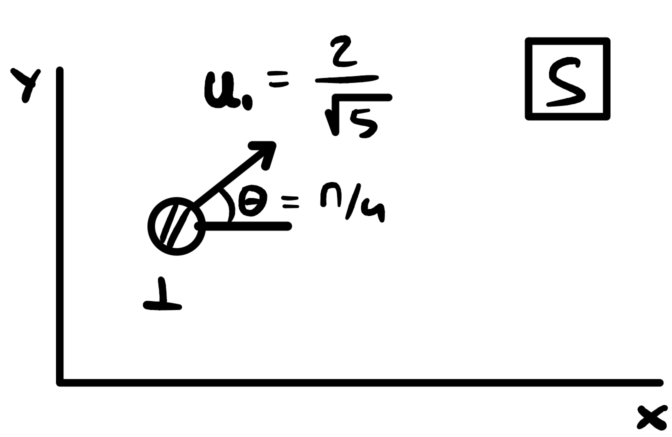
$$\gamma_1' u_{1x}' = 0 \qquad u_{1x}' = 0$$

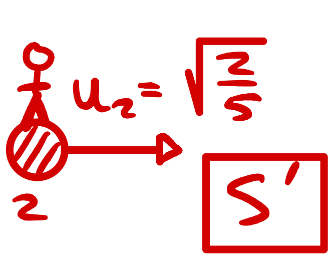
$$\gamma_1' u_{1y}' = \sqrt{2} \qquad u_{1y}' = \sqrt{2}/\sqrt{3}$$

$$\gamma_1' u_{1z}' = 0 \qquad u_{1z}' = 0$$

$$\theta' = \frac{\pi}{2} \qquad |\vec{u}_1'| = \sqrt{2}/\sqrt{3}$$

β τροπος





$$\vec{u}_{12} = \vec{u}_1'$$

u_1^{μ}	u_2^{μ}	S
$u_1^{\mu'}$	$u_2^{\mu'}$	S'

$$u_1^{\mu}\cdot u_{2\mu} = u_1^{\mu'}\cdot u_{2\mu'}$$

$$\gamma_2 = \frac{1}{\sqrt{1-2/5}} = \sqrt{5/3}$$

$$u_1^{\mu} = \left(\sqrt{5}, \sqrt{2}, \sqrt{2}, 0\right) \qquad u_1^{\mu'} = \left(\gamma_1', \gamma_1'\vec{v}_1'\right)$$

$$u_2^{\mu} = \left(\frac{\sqrt{5}}{\sqrt{3}}, \sqrt{\frac{2}{3}}, 0, 0\right) \qquad u_2^{\mu'} = \left(1, 0\right)$$

$$u_1^{\mu} \cdot u_{2\mu} = u_1^{\mu'} \cdot u_{2\mu'}$$

$$\left(-\sqrt{5}\sqrt{\frac{5}{3}} + \sqrt{2}\sqrt{\frac{2}{3}} + 0\cdot\sqrt{2} + 0\cdot 0\right) = -\gamma_1' + 0$$

$$\gamma_1' = \sqrt{3} = \frac{1}{\sqrt{1-(\vec{v}_1')^2}} =$$

$$|\vec{u}_1'|^2 = \frac{2}{3}$$

$$u_2^{\mu} = \begin{bmatrix} \sqrt{5/3} \\ \sqrt{2/3} \\ 0 \\ 0 \end{bmatrix} \qquad u_2^{\mu'} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} \sqrt{\frac{5}{3}} & -\sqrt{\frac{2}{3}} & 0 & 0 \\ -\sqrt{\frac{2}{3}} & \sqrt{\frac{5}{3}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \end{pmatrix} \begin{bmatrix} \sqrt{5/3} \\ \sqrt{2/3} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \checkmark$$