

Guppoen Einstein

$$[C^v] = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \quad [x_r] = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad y^v = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

A)

$$\text{Erwe} \quad B^v = C^v x_v$$

$$\begin{aligned} B^0 &= C^{00} x_0 + C^{01} x_1 + C^{02} x_2 + C^{03} x_3 \\ &= 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3 + 4 \cdot 4 = 30 \end{aligned}$$

$$\begin{aligned} B^1 &= C^{10} x_0 + C^{11} x_1 + C^{12} x_2 + C^{13} x_3 \\ &= 5 \cdot 1 + 6 \cdot 2 + 7 \cdot 3 + 8 \cdot 4 = 70 \end{aligned}$$

$$\begin{aligned} B^2 &= C^{20} x_0 + C^{21} x_1 + C^{22} x_2 + C^{23} x_3 \\ &= 9 \cdot 1 + 10 \cdot 2 + 11 \cdot 3 + 12 \cdot 4 = 110 \end{aligned}$$

$$\begin{aligned} B^3 &= C^{30} x_0 + C^{31} x_1 + C^{32} x_2 + C^{33} x_3 \\ &= 13 \cdot 1 + 14 \cdot 2 + 15 \cdot 3 + 16 \cdot 4 = 150 \end{aligned}$$

$$[C^v x_v] = \begin{bmatrix} 30 \\ 70 \\ 110 \\ 150 \end{bmatrix}$$

$$\text{Ergebnis } B^M = C^{kv} x_v$$

$$B^0 = C^{00} x_0 + C^{10} x_1 + C^{20} x_2 + C^{30} x_3 \\ = 1 \cdot 1 + 5 \cdot 2 + 9 \cdot 3 + 13 \cdot 4 = 90$$

$$B^1 = C^{01} x_0 + C^{11} x_1 + C^{21} x_2 + C^{31} x_3 \\ = 2 \cdot 1 + 6 \cdot 2 + 10 \cdot 3 + 14 \cdot 4 = 100$$

$$B^2 = C^{02} x_0 + C^{12} x_1 + C^{22} x_2 + C^{32} x_3 \\ = 3 \cdot 1 + 7 \cdot 2 + 11 \cdot 3 + 15 \cdot 4 = 110$$

$$B^3 = C^{03} x_0 + C^{13} x_1 + C^{23} x_2 + C^{33} x_3 \\ 4 \cdot 1 + 8 \cdot 2 + 12 \cdot 3 + 16 \cdot 4 = 120$$

$$[C^{kv} x_v] = \begin{bmatrix} 90 \\ 100 \\ 110 \\ 120 \end{bmatrix}$$

$$Y^M x_p = Y^0 x_0 + Y^1 x_1 + Y^2 x_2 + Y^3 x_3 \\ 5 \cdot 1 + 6 \cdot 2 + 7 \cdot 3 + 8 \cdot 4 = 70$$

$$x_p Y^M = x_0 Y^0 + x_1 Y^1 + x_2 Y^2 + x_3 Y^3 = 70$$

$$\begin{bmatrix} x_r & y_v \end{bmatrix} = \begin{bmatrix} 1.5 & 1.6 & 1.7 & 1.8 \\ 2.5 & 2.6 & 2.7 & 2.8 \\ 3.5 & 3.6 & 3.7 & 3.8 \\ 4.5 & 4.6 & 4.7 & 4.8 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 10 & 12 & 14 & 16 \\ 15 & 18 & 21 & 24 \\ 20 & 24 & 28 & 32 \end{bmatrix}$$

B)

$$\begin{bmatrix} n_{\mu\nu} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & I_{3 \times 3} \end{bmatrix} = n$$

$$n \cdot n^{-1} = I \rightarrow n^2 \cdot n^{-1} = n$$

$$n^2 = \begin{bmatrix} -1 & 0 \\ 0 & I_{3 \times 3} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & I_{3 \times 3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & I_{3 \times 3} \end{bmatrix} = I$$

also $n^2 \cdot n^{-1} = n \rightarrow I \cdot n^{-1} = n \rightarrow n^{-1} = n$

$$\begin{bmatrix} x_r \end{bmatrix} = \begin{bmatrix} n^{\mu\nu} & x_v \end{bmatrix}$$

$$x_0 = \cancel{n^{00}x_0} + \cancel{n^{01}x_1} + \cancel{n^{02}x_2} + \cancel{n^{03}x_3} = -x_0 = -1$$

$$x_1 = \cancel{n^{10}x_0} + \cancel{n^{11}x_1} + \cancel{n^{12}x_2} + \cancel{n^{13}x_3} = x_1 = 2$$

$$x_2 = \cancel{n^{20}x_0} + \cancel{n^{21}x_1} + \cancel{n^{22}x_2} + \cancel{n^{23}x_3} = x_2 = 3$$

$$x_3 = \cancel{n^{30}x_0} + \cancel{n^{31}x_1} + \cancel{n^{32}x_2} + \cancel{n^{33}x_3} = x_3 = 4$$

$$[x_r] = \begin{bmatrix} n_{r\nu} & x_\nu \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$[y_p] = \begin{bmatrix} n_{p\nu} & y_\nu \end{bmatrix} = \begin{bmatrix} -5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

$$[c_r^\nu] = \begin{bmatrix} n_{r\nu} & c_{\nu}^{\nu} \end{bmatrix}$$

$$c_0^0 = n_{00}^{-1} c^{00} + \cancel{n_{01} c^{10}} + \cancel{n_{02} c^{20}} + \cancel{n_{03} c^{30}} = -1$$

$$c_0^1 = n_{00}^{-1} c^{01} + \cancel{n_{01} c^{11}} + \cancel{n_{02} c^{21}} + \cancel{n_{03} c^{31}} = -2$$

$$c_0^2 = n_{00}^{-1} c^{02} + \cancel{n_{01} c^{12}} + \cancel{n_{02} c^{22}} + \cancel{n_{03} c^{32}} = -3$$

$$c_0^3 = n_{00}^{-1} c^{03} + \cancel{n_{01} c^{13}} + \cancel{n_{02} c^{23}} + \cancel{n_{03} c^{33}} = -4$$

$$c_1^0 = \cancel{n_{10} c^{00}} + n_{11}^1 c^{10} + \cancel{n_{12} c^{20}} + \cancel{n_{13} c^{31}} = 5$$

$$c_1^1 = \cancel{n_{20} c^{00}} + \cancel{n_{21} c^{10}} + n_{22}^1 c^{20} + \cancel{n_{23} c^{30}} = 9$$

$$c_1^2 = \cancel{n_{30} c^{00}} + \cancel{n_{31} c^{10}} + \cancel{n_{32} c^{20}} + n_{33}^1 c^{30} = 13$$

$$c_1^3 = \cancel{n_{10} c^0} + n_{11}^1 c^{11} + \cancel{n_{12} c^{21}} + \cancel{n_{13} c^{31}} = 6$$

$$C_{i^2} = \cancel{n_{10} C^{02}} + n_{11} \cancel{C^{12}} + \cancel{n_{12} C^{22}} + \cancel{n_{13} C^{32}} = 7$$

$$C_{i^3} = \cancel{n_{10} C^{02}} + n_{11} \cancel{C^{13}} + \cancel{n_{12} C^{23}} + \cancel{n_{13} C^{33}} = 8$$

σημείωση: Στα υπόλοιπα στοιχεία $C_{i^k j} = n_{ik} C^{kj} = \delta_{ik} C^{kj} - C^{kj}$

$$\begin{bmatrix} C_r^v \end{bmatrix} = \begin{bmatrix} n_{rk} C^{kv} \end{bmatrix} = \begin{bmatrix} -1 & -2 & -3 & -4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} = n \cdot C$$

$$\begin{bmatrix} C^r_v \end{bmatrix} = \begin{bmatrix} C^{rk} n_{kv} \end{bmatrix} = C \cdot n$$

για εντοπίσιμη λύση σε μορφή γιγάντιου πινακών

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} -1 & 2 & 3 & 4 \\ -5 & 6 & 7 & 8 \\ -9 & 10 & 11 & 12 \\ -13 & 14 & 15 & 16 \end{bmatrix} = \begin{bmatrix} C^r_v \end{bmatrix}$$

$$[C_{\mu\nu}] = [n_{\mu e} C^{\mu e} n_{e\nu}] = n \cdot C \cdot n$$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 & 4 \\ -5 & 6 & 7 & 8 \\ -9 & 10 & 11 & 12 \\ -13 & 14 & 15 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & -3 & -4 \\ -5 & 6 & 7 & 8 \\ -9 & 10 & 11 & 12 \\ -13 & 14 & 15 & 16 \end{bmatrix} = [C_{\mu\nu}]$$