$A'^{\mu} = (A^0, A^1, 0, 0)$

Για $\quad A^{\mu} = (A^0, A^1, 0, 0)$

δια συντομία παραλείπουμε τις y, z διαστάσεις

1) $A^{\mu} = \begin{pmatrix} A^0 \\ A^1 \end{pmatrix}$

$$A_{\mu} = \eta_{\mu\nu} A^{\nu} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A^0 \\ A^1 \end{pmatrix} = \begin{pmatrix} -A^0 \\ A^1 \end{pmatrix}$$

2) $$A'^{\mu} = \begin{pmatrix} \gamma & -\gamma\upsilon \\ -\gamma\upsilon & \gamma \end{pmatrix} \begin{pmatrix} A^0 \\ A^1 \end{pmatrix} = \begin{bmatrix} \gamma(A^0 - \upsilon A^1) \\ \gamma(A^1 - \upsilon A^0) \end{bmatrix}$$

3) $$A'_{\mu} = \eta_{\mu\kappa} A'^{\kappa} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} A'^0 \\ A'^1 \end{pmatrix} =$$

$A^{\mu} = \eta^{\mu\kappa} A_{\kappa}$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \overbrace{\begin{pmatrix} \gamma & -\gamma\upsilon \\ -\gamma\upsilon & \gamma \end{pmatrix}}^{\Lambda(\vec{\upsilon})} \begin{pmatrix} A^0 \\ A^1 \end{pmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \gamma & -\gamma\upsilon \\ -\gamma\upsilon & \gamma \end{pmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} -A^0 \\ A^1 \end{pmatrix}$$

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άρα $$A'_{\mu} = \begin{pmatrix} \gamma & \gamma\upsilon \\ \gamma\upsilon & \gamma \end{pmatrix} A_{\mu}$$

$$A'^{\mu} = \begin{pmatrix} \gamma & -\gamma\upsilon \\ -\gamma\upsilon & \gamma \end{pmatrix} A^{\mu}$$

4) $A^{\nu} A_{\nu} = -(A^0)^2 + (A^1)^2$

5) $A'^{\mu} A'_{\mu} = -(A'^0)^2 + (A'^1)^2$

$$= -\left(\gamma(A^0 - vA^1)\right)^2 + \left(\gamma(A^1 - vA^0)\right)^2$$

$$= -\gamma^2(A^0)^2 - \gamma^2 v^2 (A^1)^2 + 2\gamma A^0 v A^1 + \gamma^2 (A^1)^2 + \gamma^2 v^2 (A^0)^2 - 2\gamma A^1 v A^0$$

$$= \left[\gamma^2(1-v^2)\right](A^1)^2 - \left[\gamma^2(1-v^2)\right](A^0)^2$$

$$= -(A^0)^2 + (A^1)^2$$