ενεργεια ορμη μαζα

$$p^\mu = m u^\mu = m\frac{dx^\mu}{d\tau} = m(\gamma, \gamma\vec{u}) = \underbrace{\begin{bmatrix} m\gamma \\ m\gamma\vec{u}\end{bmatrix}}_{m>0} = \begin{bmatrix} E \\ \vec{p}\end{bmatrix} = \begin{pmatrix}\sqrt{|\vec{p}|^2+m^2} \\ \vec{p}\end{pmatrix}$$

δουλευει και για $m=0$

$d\tau^2 = dt^2 - dx^2 - dy^2 - dz^2 = dt^2(1-u^2) = dt^2\gamma^{-2}$

$\gamma\, d\tau = dt \rightsquigarrow \frac{d(\ldots)}{d\tau} = \gamma\frac{d(\ldots)}{dt}$

$x^\mu = (t,x,y,z)$

$\frac{dx^\mu}{d\tau} = \gamma\frac{d(t,x,y,z)}{dt} = \gamma\left(1, \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$

$\vec{u} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) \qquad u^\mu = (\gamma, \gamma\vec{u})$

Κ.Μ. $\quad \vec{p} = m\vec{u}$

Σ.Μ. $\quad \vec{p} = \gamma m\vec{u} = \gamma m(u_x, u_y, u_z)$

$p^\mu p_\mu = +m^2 \quad (+ - - -)$

$p^\mu p_\mu = -m^2 \quad (- + + +)$

$E = \gamma m = \left(m + \frac{p^2}{2m} + \ldots\right)$

$\boxed{T \equiv E - m}$

$\boxed{E^2 = p^2 + m^2} \qquad p = |\vec{p}|$

	$m>0$	$m=0$
p^μ	$\begin{pmatrix} m\gamma \\ m\gamma\vec{u}\end{pmatrix} = \begin{pmatrix}\sqrt{m^2+p^2} \\ \vec{p}\end{pmatrix}$	$\begin{pmatrix} p \\ \vec{p}\end{pmatrix} = \begin{pmatrix} E \\ E\hat{n}\end{pmatrix}$

$p^\mu(e^-)\cdot p_\mu(e^-) = -(511\,\mathrm{keV})^2$

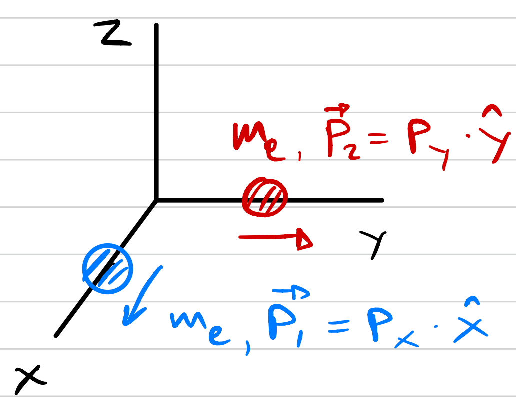
$p^\mu(p^+)\cdot p_\mu(p^+) = -(0.938\,\mathrm{GeV})^2$

$m_{p^+} = 0.938\,\mathrm{GeV} \simeq 1\,\mathrm{GeV}$

$p^\mu(\gamma)\cdot p_\mu(\gamma) = 0$

$(- + + +)$

τετραορμη e^-



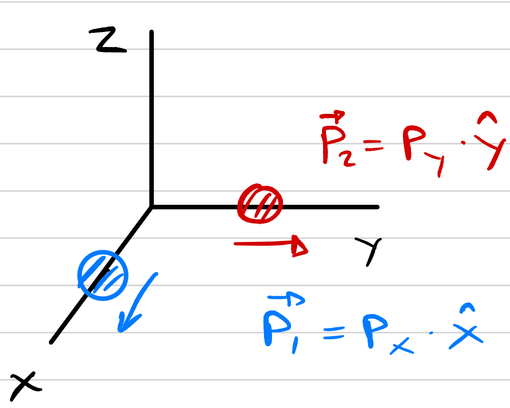
$P_1^\mu = \left(\sqrt{m_e^2 + P_x^2}, P_x, 0, 0\right)$

$P_2^\mu = \left(\sqrt{m_e^2 + P_y^2}, 0, P_y, 0\right)$

$p^2 = -m_e^2 \quad (- + + +)$

$p^2 = m_e^2 \quad (+ - - -)$

τετραορμη φωτονιου



$P_1^\mu = \left(\sqrt{P_x^2}, P_x, 0, 0\right) = (E_1, E_1, 0, 0)$

$P_2^\mu = \left(\sqrt{P_y^2}, 0, P_y, 0\right) = (E_2, 0, E_2, 0)$

$p^2 = 0$

$E = |\vec{p}| = h\nu \overset{c=\lambda\nu}{=} h\frac{c}{\lambda}$

	στο SI
$[h]$ = ενεργεια × χρονος	J · s
$[\nu]$ = συχνοτητα	s^{-1}

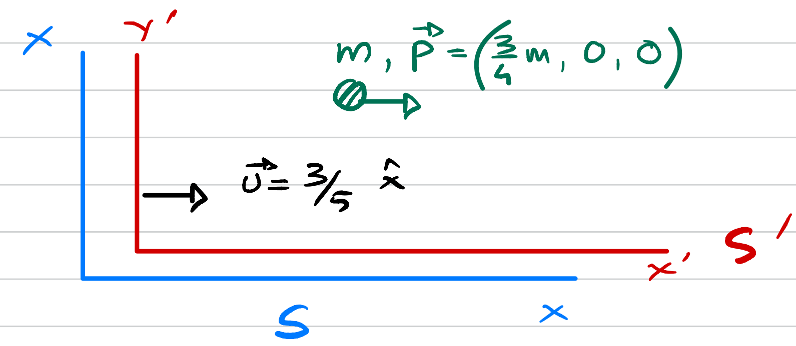
μετασχηματισμοί Lorentz

$p^{\mu'} = \Lambda^{\mu'}{}_{\mu}\, p^{\mu}$ | $\underset{\boxed{S'}}{P'} = \Lambda \cdot \underset{\boxed{S}}{P}$

$p^{\mu} = \left(\frac{5}{4}m, \frac{3}{4}m, 0, 0\right)$

$p^{\mu'} = (E', \vec{p}\,')$

$\Lambda\left(\vec{v} = \frac{3}{5}\hat{x}\right)$



E' = ?

a)	5/4 m	44%
b)	2m	7%
c)	m	31%
d)	$\frac{3}{5}m$	18%

$$\vec{v} = \frac{\vec{p}}{E} = \frac{\gamma m \vec{v}}{\gamma m} = \frac{3}{5}\hat{x}$$

$$\gamma = \frac{1}{\sqrt{1-(3/5)^2}} = 5/4$$

$\boxed{S}$: $E^2 = p^2 + m^2$

$\boxed{S'}$: $(E')^2 = m^2$

$$\begin{pmatrix} \gamma & -\gamma v & 0 & 0 \\ -\gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} E \\ P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} 5/4 & -3/4 & 0 & 0 \\ -3/4 & 5/4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 5m/4 \\ 3m/4 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} m \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

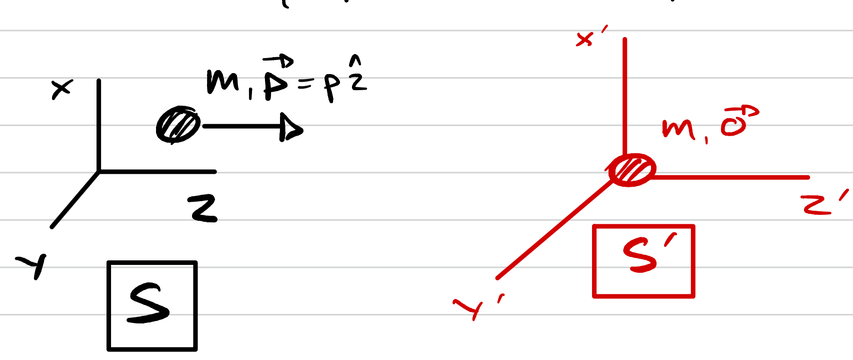
τετραορμή ενός σωματιδίου στο σύστημα ηρεμίας του

παράδειγμα

$\boxed{S}$: $p^{\mu} = (E, 0, 0, P)$

$\boxed{S'}$: $p^{\mu'} = (m, 0, 0, 0)$

ποια προώθηση χρειαζόμαστε για να μεταβούμε στο σύστημα ηρεμίας του σωματιδίου;



$\boxed{\vec{v} = \frac{\vec{p}}{E}}$ $\boxed{\gamma = \frac{E}{m}}$

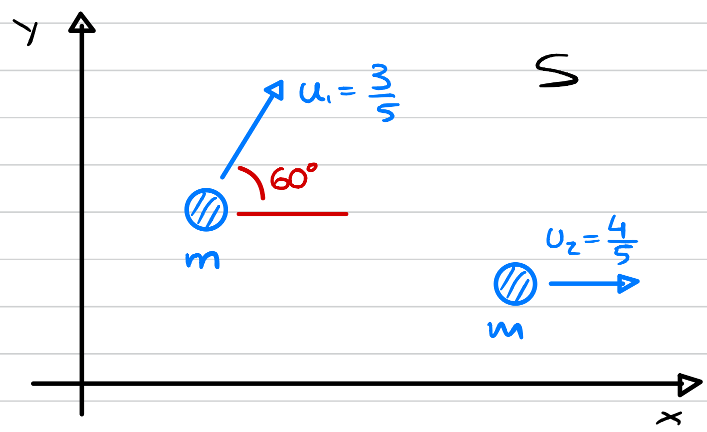
$|\vec{p}| = p$ $\vec{p} = \gamma m \vec{v}$ $m^2 = E^2 - p^2$

$p^{\mu'} = \Lambda^{\mu'}{}_{\mu}\, p^{\mu} \longrightarrow$

$$p^{\mu'} = \begin{pmatrix} \gamma & 0 & 0 & -\gamma v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma v & 0 & 0 & \gamma \end{pmatrix}\begin{pmatrix} E \\ 0 \\ 0 \\ P \end{pmatrix} = \begin{bmatrix} \gamma E - \gamma v P \\ 0 \\ 0 \\ -\gamma v E + \gamma P \end{bmatrix}$$

$$= \begin{bmatrix} \frac{E}{m}E - \frac{E}{m}\frac{P}{E}P \\ 0 \\ 0 \\ -\frac{E}{m}\frac{P}{E}E + \frac{E}{m}P \end{bmatrix} = \begin{bmatrix} (E^2-p^2)/m^2 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} m \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

προβλημα 1 (2021)



S' = συστημα ηρεμιας του "2"

a) $E_i, \vec{P}_i, T_i$ στα S, S'

b) η ταχυτητα του "1" στο συστημα ηρεμιας του 2

$\cos 60 = \frac{1}{2} \qquad \sin 60 = \sqrt{3}/2$

$$u_1 = \frac{3}{5}(\cos\theta, \sin\theta, 0) = \left(\frac{3}{10}, \frac{3\sqrt{3}}{10}, 0\right) \qquad \gamma_1 = \frac{5}{4} \qquad P_1^\mu = (m\gamma_1, m\gamma_1 \vec{u}_1)$$

$$u_2 = \left(\frac{4}{5}, 0, 0\right) \qquad \gamma_2 = \frac{5}{3} \qquad P_2^\mu = (m\gamma_2, m\gamma_2 \vec{u}_2)$$

$$P_1^\mu = m\begin{pmatrix} 5/4 \\ 3/8 \\ 3\sqrt{3}/8 \\ 0 \end{pmatrix} \qquad P_2^\mu = m\begin{pmatrix} 5/3 \\ 4/3 \\ 0 \\ 0 \end{pmatrix} \qquad \Lambda = \begin{pmatrix} 5/3 & -4/3 & 0 & 0 \\ -4/3 & 5/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$P_1^2 = -\frac{25}{16} + \frac{9}{64} + \frac{27}{64} = -\frac{100}{64} + \frac{36}{64} = -1 \qquad P_2^2 = -\frac{25}{9} + \frac{16}{9} = -1 \quad [GeV^2]$$

$$P_2^{\mu'} = m\begin{pmatrix} 5/3 & -4/3 & 0 & 0 \\ -4/3 & 5/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 5/3 \\ 4/3 \\ 0 \\ 0 \end{pmatrix} = m\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \vec{u}_i' = \frac{\vec{P}_i'}{E_i}$$

"ωμος"

$$P_1^{\mu'} = m\begin{pmatrix} 5/3 & -4/3 & 0 & 0 \\ -4/3 & 5/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 5/4 \\ 3/8 \\ 3\sqrt{3}/8 \\ 0 \end{pmatrix} = m\begin{pmatrix} 19/12 \\ -25/24 \\ 3\sqrt{3}/8 \\ 0 \end{pmatrix}$$

$$\vec{u}_1' = \frac{\vec{P}_1'}{E_1'} = \frac{[-25, 9\sqrt{3}, 0]^T}{38}$$

$$|\vec{u}_1'| \simeq 0.78 \qquad \vec{u}_{12}' = \vec{u}_1'$$

$E_1' = \frac{19}{12}m \qquad E_1 = \frac{5}{4}m$ διατηρηση της ενεργειας ?

"κομψος" $P_1^\mu \cdot P_{2\mu} = P_1^{\mu'} P_{2\mu'} = \mathbf{P}_1 \cdot \mathbf{P}_2$ = αναλλοιωτο Lorentz

$$P_1^\mu \cdot P_{2\mu} = m^2\left[-\frac{5}{4}\cdot\frac{5}{3} + \frac{3}{8}\cdot\frac{4}{3}\right] = m^2\left[-\frac{25}{12} + \frac{6}{12}\right] = -m^2\frac{19}{12}$$

$$\gamma_{12} = 19/12 = \frac{1}{\sqrt{1-u_{12}^2}}$$

$\boxed{\text{ξερω οτι στο } S'}$

$$P_2^{\mu'} = m(1, 0, 0, 0)$$
$$P_{1\mu'} = m(-\gamma_{12}, \gamma_{12}\vec{u}_{12})$$

αρα $\rightarrow P_1^{\mu'} \cdot P_{2\mu'} = -m^2\gamma_{12}$

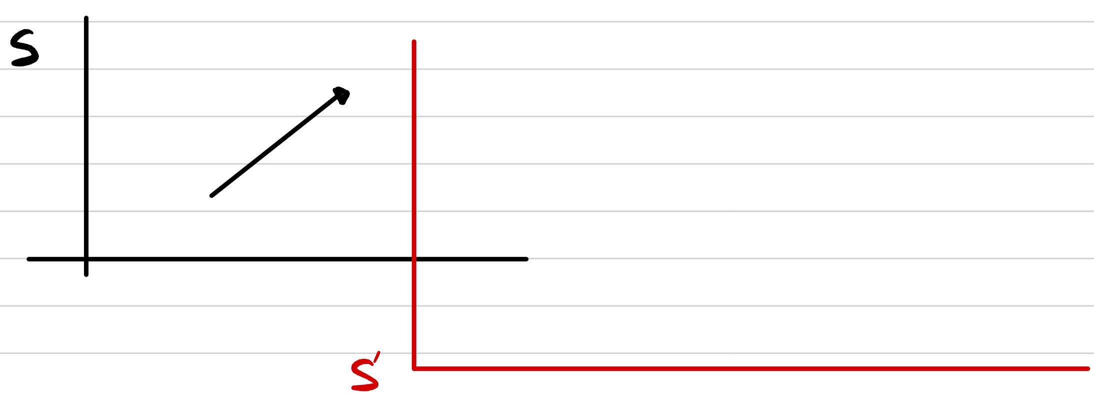
$$u_{12} = \sqrt{(1-\gamma^{-2})} \simeq 0.78$$

$\mathbf{P}_1 \cdot \mathbf{P}_2 \qquad \mathbf{P}_1^2 \qquad \mathbf{P}_2^2$

''ασχημος''

μετασχηματισμος $u=\left(\frac{dx}{dt},\frac{dy}{dt},\frac{dz}{dt}\right) \longrightarrow u'=\left(\frac{dx'}{dt'},\frac{dy'}{dt'},\frac{dz'}{dt'}\right)$

αφου η προωθηση ειναι // $\hat{x}$ περιμενουμε το $u_y' = u_y$?



$$dx' = \gamma(dx - u\,dt)$$
$$dt' = \gamma(dt - u\,dx)$$

$$\frac{dx_1'}{dt_1'} = \frac{\gamma(dx_1 - u\,dt_1)}{\gamma(dt_1 - u\,dx_1)} = \frac{u_{1x} - u}{1 - u_{1x}\cdot u} = \frac{3/10 - 4/5}{1 - 3/10\cdot 4/5} = -\frac{25}{38}$$

$$\frac{dy_1'}{dt_1'} = \frac{dy_1}{\gamma(dt_1 - u\,dx_1)} = \frac{u_{1y}/\gamma}{1 - u_{1x}\cdot u} = \frac{3\sqrt{3}/10 \cdot \frac{3}{5}}{38/50} = \frac{9\sqrt{3}}{38}$$

$$\frac{dz_1'}{dt_1'} = \frac{dz_1}{\gamma(dt_1 - u\,dx_1)} = \varnothing$$

$$\vec{v}_1 = \left(\frac{3}{10}, \frac{3\sqrt{3}}{10}, 0\right)$$

$$\gamma = 5/3 \qquad u = 4/5$$

ερωτησεις απο το προηγουμενο μαθημα

1) γιατι $p^\mu = m u^\mu$?

2) γιατι $u^\mu = \frac{dx^\mu}{d\tau}$ και οχι $\frac{dx^\mu}{dt}$?

3) γιατι η πλειοψηφια ειπε οτι $E' = E$ οταν ?

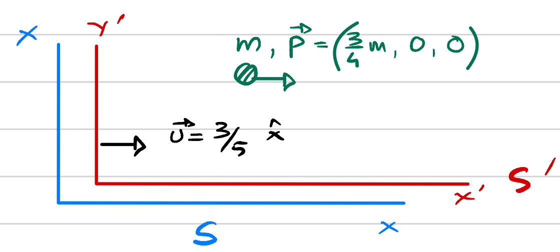
$p^\mu = \left(\frac{5}{4}m, \frac{3}{4}m, 0, 0\right)$

$p^{\mu'} = (E', \vec{p}\,')$

$p^{\mu'} = \Lambda^{\mu'}{}_{\mu}\, p^\mu \quad | \quad \underset{\boxed{S'}}{P'} = \Lambda \cdot \underset{\boxed{S}}{P}$

$\Lambda\left(\vec{v} = \frac{3}{5}\hat{x}\right)$

$E' = ?$		
a) $5/4\, m$	44%	!!!
b) $2m$	7%	
c) m	31%	
d) $\frac{3}{5} m$	18%	



παραδειγμα

(1) $m_1 v_1 + m_2 v_2 = m_3 v_3 + m_4 v_4$

(2) $\gamma_1 m_1 v_1 + \gamma_2 m_2 v_2 = \gamma_3 m_3 v_3 + \gamma_4 m_4 v_4$ S

(2): $(P_{1x}) \quad (P_{2x}) \quad (P_{3x}) \quad (P_{4x})$

---

(1)' $m_1 v_1' + m_2 v_2' \stackrel{?}{=} m_3 v_3' + m_4 v_4'$

(2)' $\frac{m_1 v_1'}{\sqrt{1-(v_1')^2}} + \gamma_2' m_2 v_2' \stackrel{?}{=} \gamma_3' m_3 v_3' + \gamma_4' m_4 v_4'$ S'

$m_1 v_1'$

$$(1)' \to m_1 \frac{v_1 - v}{1 - v_1 \cdot v} + m_2 \frac{v_2 - v}{1 - v_2 \cdot v} \stackrel{?}{=} m_3 \frac{v_3 - v}{1 - v_3 \cdot v} + m_4 \frac{v_4 - v}{1 - v_4 \cdot v}$$

$$(2)' \to P_{1x}' + P_{2x}' \stackrel{?}{=} P_{3x}' + P_{4x}'$$

$$\begin{pmatrix} E_i' \\ P_{xi}' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v \\ -\gamma v & \gamma \end{pmatrix} \begin{pmatrix} E_i \\ P_{xi} \end{pmatrix} \qquad S' = \Lambda S \qquad \gamma = \frac{1}{\sqrt{1-v^2}}$$

$$P_{1x} - vE_1 + P_{2x} - vE_2 = P_{3x} - vE_3 + P_{4x} - vE_4$$

$$\cancel{P_{1x} + P_{2x}} - v(E_1 + E_2) = \cancel{P_{3x} + P_{4x}} - v(E_3 + E_4)$$

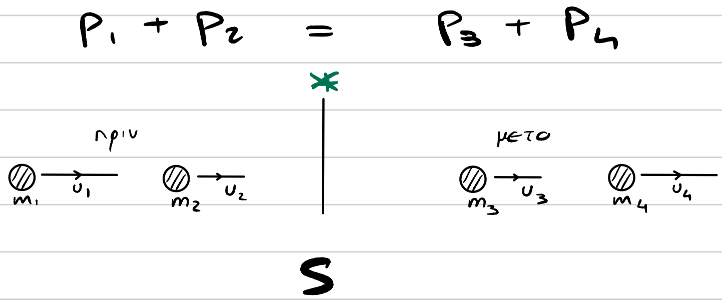
$$E_1 + E_2 = E_3 + E_4$$

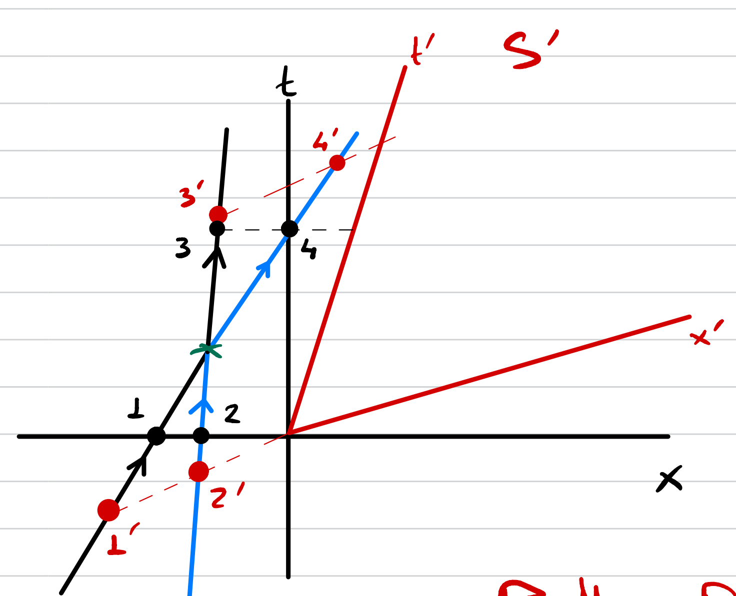
μετ. Γαλιλαίου

(1) $\to v_i' = v_i - v$

$m_1 + m_2 = m_3 + m_4$

$$P_1 + P_2 = P_3 + P_4$$





$$P_1^\mu + P_2^\mu = P^\mu$$

$$P^\mu \to P^{\mu'} = \Lambda^{\mu'}{}_{\nu} P^\mu$$

Ελεγχος του $E=\frac{1}{2}m\upsilon^2$ σε γραμμικο επιταχυντη

(W. Bertozzi "The ultimate speed" 1962)

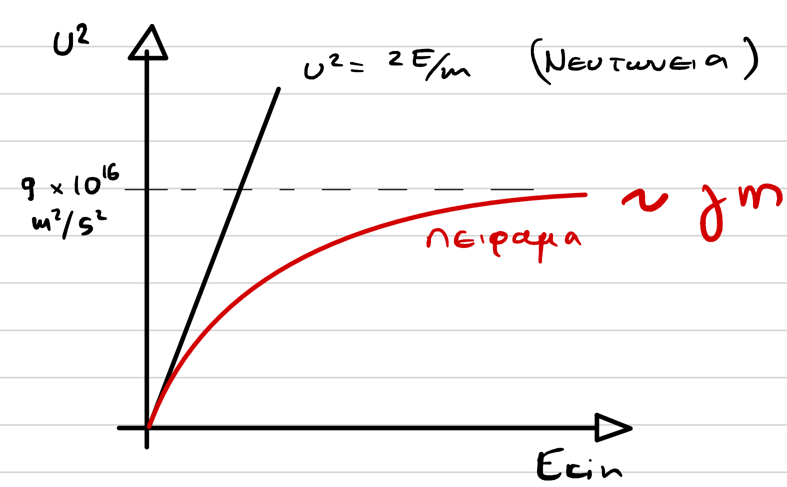
$\upsilon^2 = \frac{2E}{m}$

$m_e = 511\ keV$ → το πιο ελαφρυ (ελευθερο) φορτισμενο σωματιδιο

$$\upsilon = \frac{\Delta x}{\Delta t} = \frac{x_2-x_1}{t_2-t_1} \simeq \frac{8.4\,m}{\Delta t}$$

$$E_{kin} = q\Delta V = E_{cal} \pm \Delta E_{cal}$$

E_{kin} [MeV]	Δt [ns]	υ [$10^8 \frac{m}{s}$]
0.5	32.3	2.60
1.0	30.8	2.73
1.5	29.2	2.88
4.5	28.4	2.99
15	28.0	3.00



Νευτωνεια : $E_{kin} = \left(\frac{1}{2}m\right)\upsilon^2$

ΕΘΣ : $E_{kin} = (\gamma-1)m = \frac{1}{2}m\upsilon^2 + \frac{3}{8}m\upsilon^4 + \ldots$

$$p^0 \equiv E = \frac{m}{\sqrt{1-\upsilon^2}} = \left(m + \frac{1}{2}m\upsilon^2 + \frac{3}{8}m\upsilon^4 + \ldots\right) = m + T$$

$f(x) = (1-x)^{-1/2}$

$f'(x) = \frac{1}{2}(1-x)^{-3/2}$

$f''(x) = \frac{3}{4}(1-x)^{-5/2}$

$(1-x)^{-1/2} = 1 + \frac{1}{2}x + \frac{1}{2!}\frac{3}{4}x^2 + \ldots$

A.P. French: "Special Theory of Relativity"

Youtube: "The Ultimate Speed" (Bertozzi, 1962)

Τετραορμη συστηματος σωματιδιων

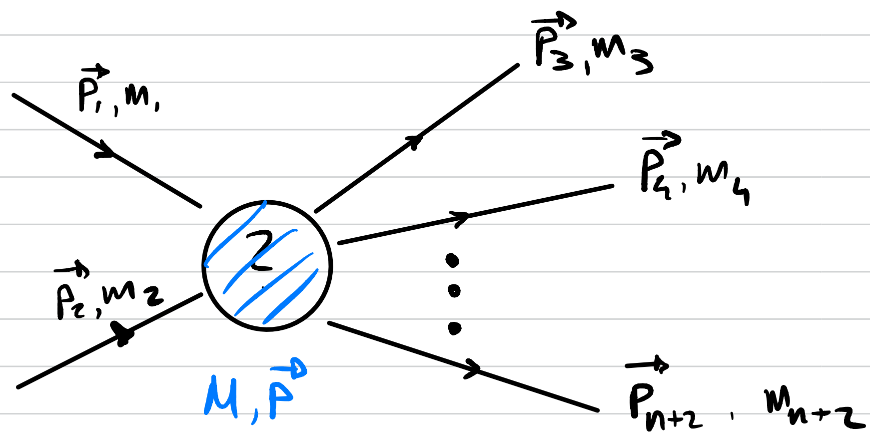
$$P^\mu = p_1^\mu + p_2^\mu + \dots p_m^\mu \qquad (m \text{ σωματιδια})$$

αναλλοιωτη μαζα συστηματος $= \sqrt{-P^\mu P_\mu}$ $(-+++)$

$\sqrt{P^\mu P_\mu} = \sqrt{s}$ για $(+---)$

$\sqrt{s}$ = ενεργεια στο κεντρο μαζας (ορμης)

$2 \to n$



$m_1 + m_2 \neq m_3 + m_4 + \dots m_{n+2}$

$$\begin{cases} E_1 + E_2 = E_3 + E_4 + \dots E_{n+2} \\ \vec{p}_1 + \vec{p}_2 = \vec{p}_3 + \vec{p}_4 + \dots \vec{p}_{n+2} \end{cases}$$

$$\sum_{i=1}^{2} \begin{pmatrix} E_i \\ \vec{p}_i \end{pmatrix} = \sum_{i=3}^{n+2} \begin{pmatrix} E_i \\ \vec{p}_i \end{pmatrix}$$

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu + \dots p_{n+2}^\mu$$

$$\boxed{\text{Η συνολικη τετραορμη ενος κλειστου συστηματος S ειναι σταθερη}}$$

$$P^\mu \equiv p_1^\mu + p_2^\mu = \left(\sqrt{M^2 + |\vec{P}|^2}, \vec{P}\right)$$

$\vec{P}$ = 3-ορμη του συστηματος S

M = αναλλοιωτη μαζα του συστηματος S

$M = \sqrt{s}$! $\qquad M \neq m_1 + m_2$!

Συστημα κεντρου μαζας (ορμης) ≡ οταν $\vec{P} = 0 = \sum_i \vec{p}_i$

Βοηθαει να σκεφτομαστε το P^μ σαν ενα υποθετικο σωματιδιο "CM" το οποιο κουβαλα την ενεργεια & ορμη ολοκληρου του συστηματος P^μ_{cm}

μετασχηματισμος Lorentz προς το K.O.

$P^\mu_{cm} \equiv P^\mu = (E, \vec{P}) \quad \longrightarrow \quad P^{\mu'} \equiv (M, \vec{0})$ στο κεντρο ορμης

$\boxed{S} \longrightarrow \boxed{S'}$

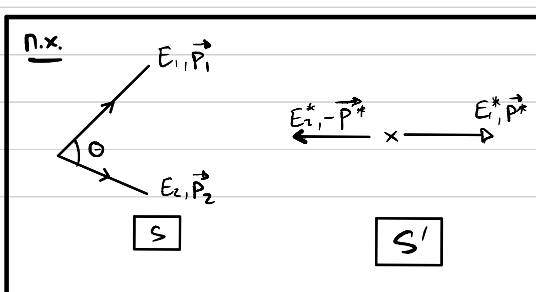
$P^\mu \to P^{\mu'} = \Lambda^{\mu'}{}_\nu P^\nu \qquad \Lambda(\vec{v}_{cm}) = ?$

$$\vec{v}_{cm} = \frac{\vec{P}}{E} = \frac{\gamma_{cm} M \vec{v}_{cm}}{\gamma_{cm} M}$$

$$\gamma_{cm} = \frac{E}{M} = \frac{1}{\sqrt{1 - v_{cm}^2}}$$

$$E = \sqrt{M^2 + |\vec{P}|^2}$$

π.χ.



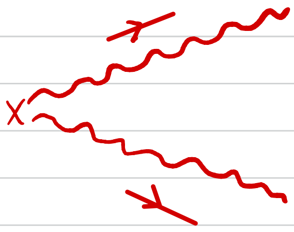
$$\vec{v}_{cm} = \frac{\vec{p}_1 + \vec{p}_2}{E_1 + E_2}$$

$$\gamma_{cm} = \frac{E}{M} = \frac{E_1 + E_2}{\sqrt{(E_1+E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2}}$$

μαζα συστηματος δυο φωτονιων

$$m_{sys} = \begin{cases} 0 & A \\ >0 & B \\ \text{εξαρταται} & C \end{cases}$$

A	37 %
B	29 %
C	34 %



$\vec{p}_1 = (3,4,0)$

$\vec{p}_2 = (3,-4,0)$

$p_1^\mu = (5,3,4,0)$ $\quad p_2^\mu = (5,3,-4,0)$

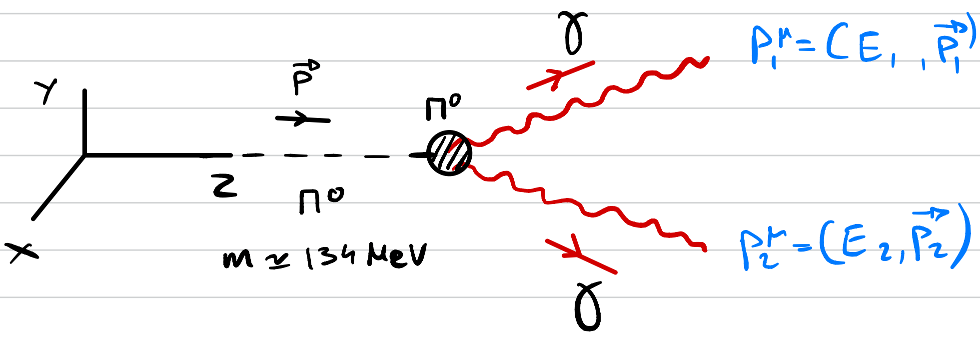
$$(p_1+p_2)^\mu = (10,6,0,0) = p^\mu$$

$$\sqrt{s} = M = \sqrt{10^2-6^2} = 8$$

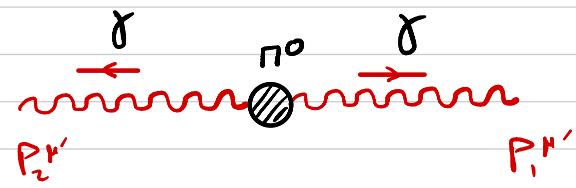
$\vec{p}_1 = (3,4,0)$

$\vec{p}_2 = (-3,-4,0)$

$p^\mu = (10,\vec{0})$ $\qquad \sqrt{s} = 10$



$\boxed{S}$



ΚΟ

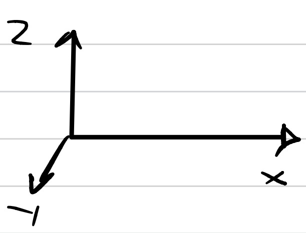
$\boxed{S'}$

αναλλοιωτες ποσοτητες:

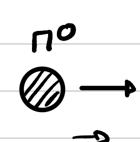
$$p_1^2 = p_2^2 = (p_1')^2 = (p_2')^2 = 0$$

$$(p_1+p_2)^2 = (p_1'+p_2')^2 = -m_{\pi^0}^2 \qquad (-+++)$$

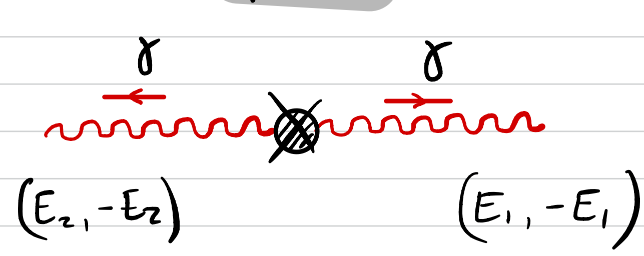
$$= m_{\pi^0}^2 = \sqrt{s} \qquad (+---)$$



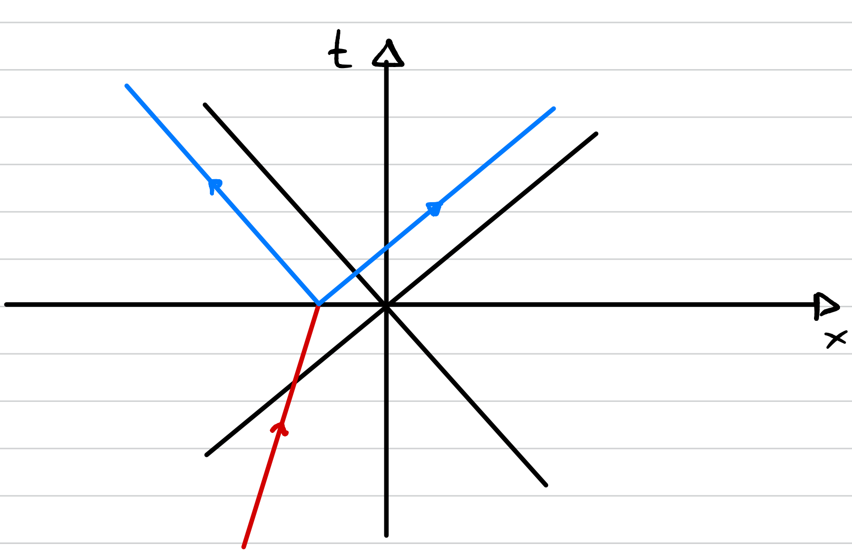
πριν



μετα



$E_1 > E_2 \qquad (E_1 - E_2)\hat{x} = \vec{p}$



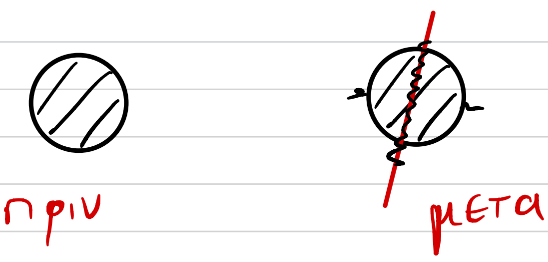
$$\begin{array}{cccc} \pi^0 & & \gamma & & \gamma \\ \begin{pmatrix} \sqrt{m^2+p^2} \\ \vec{p} \end{pmatrix} & = & \begin{pmatrix} E_1 \\ E_1 \end{pmatrix} & + & \begin{pmatrix} E_2 \\ -E_2 \end{pmatrix} \\ \text{πριν} & & & \text{μετα} & \end{array}$$

παραδειγματα αντιδρασεων

1 → 2

($M = m_1 + m_2$ νευτωνεια)

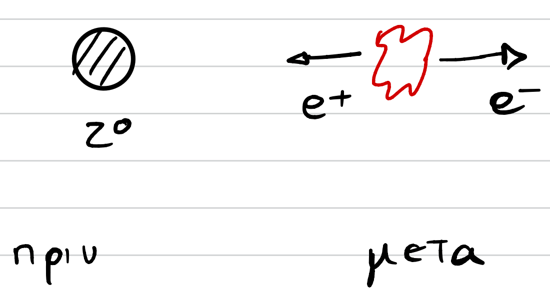
Νευτωνια στο Κ.Ο.



τα δυο κομματια παραμενουν ακινητα διοτι $M = m_1 + m_2$

$Z^0 \to e^- e^+$ $m_Z = 91$ GeV $m_{e^-} = m_{e^+} = 511$ keV

$m_Z / m_e \simeq 178$k



μαζα των προϊοντων μικροτερη της αρχικης !

$M > m_{e^-} \; m_{e^+}$

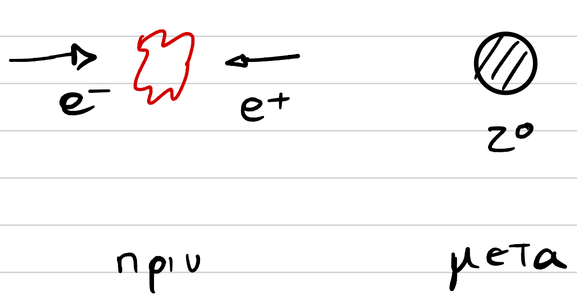
τα $e^\pm$ εχουν μεγαλη ταχυτητα ακομη και στο Κ.Ο.

$p^* \simeq M/2$

μπορει η αλληλεπιδραση να λαβει χωρα με την αντιθετη χρονικη σειρα ?

T: { συμμετρια αντιστροφης χρονου } ~ διατηρηση της ενεργειας (Nother)

2 → 1



συμμετρια αντιστροφης χρονου

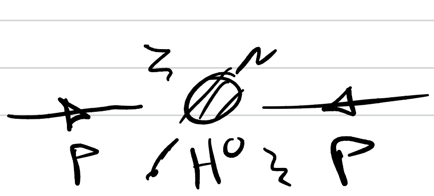
$m_{e^-} + m_{e^+} \leq M_Z$

μαζα του προϊοντος ειναι μεγαλυτερη της αρχικης !

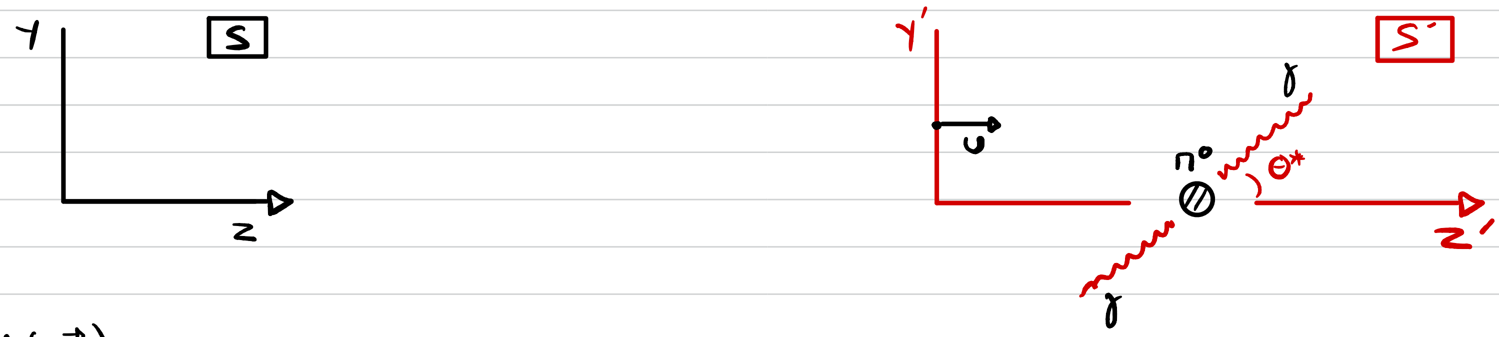
CERN

$pp \to H^0 + (\ldots)$ Higgs boson

Large Hadron Collider



προβλημα 2 (2021)



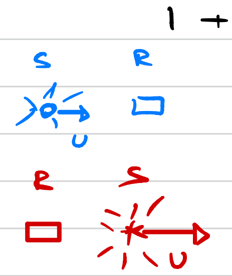
$$\Lambda(-\vec{\upsilon})$$

$$\begin{bmatrix} \gamma & 0 & 0 & \gamma\upsilon \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\upsilon & 0 & 0 & \gamma \end{bmatrix} \quad \Bigg| \quad m\begin{bmatrix}1\\0\\0\\0\end{bmatrix} = \frac{m}{2}\begin{bmatrix}1\\ \sin\theta^*\\0\\ \cos\theta^*\end{bmatrix} + \frac{m}{2}\begin{bmatrix}1\\ -\sin\theta^*\\0\\ -\cos\theta^*\end{bmatrix} \qquad \boxed{S'}$$

$$\begin{bmatrix} \gamma & 0 & 0 & \gamma\upsilon \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\upsilon & 0 & 0 & \gamma \end{bmatrix} \frac{m}{2}\begin{bmatrix}1\\ \sin\theta^*\\0\\ \cos\theta^*\end{bmatrix} = \begin{bmatrix} \frac{\gamma m}{2}(1+\upsilon\cos\theta^*) \\ \frac{m}{2}\sin\theta^* \\ 0 \\ \frac{\gamma m}{2}(\upsilon+\cos\theta^*) \end{bmatrix} = E\begin{bmatrix}1\\ \sin\theta\\0\\ \cos\theta\end{bmatrix}$$

$$E = \frac{\gamma m}{2}(1+\upsilon\cos\theta^*) \quad \Big| \quad \sin\theta = \frac{\sin\theta^*}{\gamma(1+\upsilon\cos\theta^*)} \qquad \cos\theta = \frac{\upsilon+\cos\theta^*}{1+\upsilon\cos\theta^*}$$

$$\boxed{f = \gamma(1+\upsilon\cos\theta^*)\, f^*}$$



blue shift
κυανη μετατοπιση

red shift
ερυθρη μετατοπιση

διαμηκες Doppler

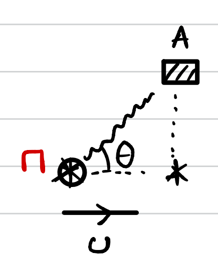
$\theta^* = 0$ $\qquad f = \sqrt{\dfrac{1+\upsilon}{1-\upsilon}}\, f^* \quad \Big| \quad \theta = 0$

$\theta^* = \frac{\pi}{2}$ $\qquad f = \gamma f^*$ Blue shift

$\cos\theta = \upsilon$

↑ συχνοτητα στον ανιχνευτη για ακτινα με $\theta^* = \pi/2$ στο $\boxed{S'}$

#1

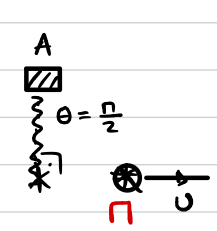


Πηγη Ανιχνευτης

$\theta = \pi/2 \rightarrow \cos\theta^* = -\upsilon$

$$f = \gamma(1-\upsilon^2)\, f^* = \sqrt{1-\upsilon^2}\, f^*$$

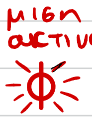
#2



$f = f^*/\gamma$ red shift → συχνοτητα στον ανιχνευτη για ακτινα με $\theta = \pi/2$ στο $\boxed{S}$

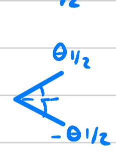
$\theta^* = \pm\frac{\pi}{2}$

μιση ακτινοβολια



$\boxed{S'}$

$\theta_{1/2}$

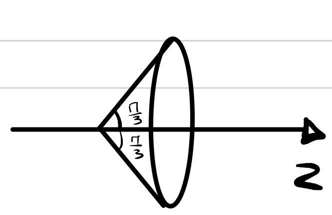


$\boxed{S}$

$$\sin\theta_{1/2} = \frac{1}{\gamma} \qquad \cos\theta_{1/2} = \upsilon$$

$$\tan(\theta_{1/2}) = (\gamma\upsilon)^{-1}$$

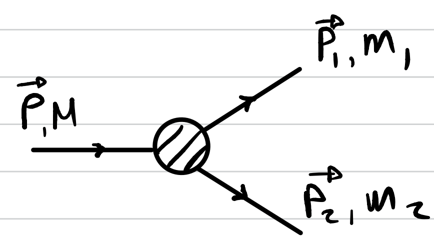
π.χ. για $\upsilon = \frac{1}{2}$ $\qquad \theta_{1/2} = 60^\circ$



$\boxed{S}$

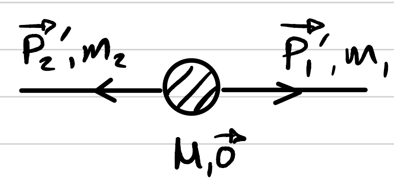
$1 \to 2$ γενικη περιπτωση

$M \to m_1 \, m_2$



$\boxed{S}$

κεντρο μαζας (ορμης)



$\boxed{S'}$

$P = p_1 + p_2$ $\boxed{S}$

$P' = p_1' + p_2'$ $\boxed{S'}$

αλλά $P \neq P' = \Lambda P$

ομως $P^2 = (P')^2$

π.χ.

$P = (E, \vec{P})$
$P_1 = (E_1, \vec{P}_1)$
$P_2 = (E_2, \vec{P}_2)$

$\boxed{S}$

$P' = (M, \vec{0})$
$P_1' = (E_1^*, \vec{p}^*)$
$P_2' = (E_2^*, -\vec{p}^*)$

$\boxed{S'}$

$P' \neq p_1 + p_2$ ομως $(P')^2 = (p_1 + p_2)^2$

$M^2 = (E_1 + E_2)^2 - (\vec{P}_1 + \vec{P}_2)^2$

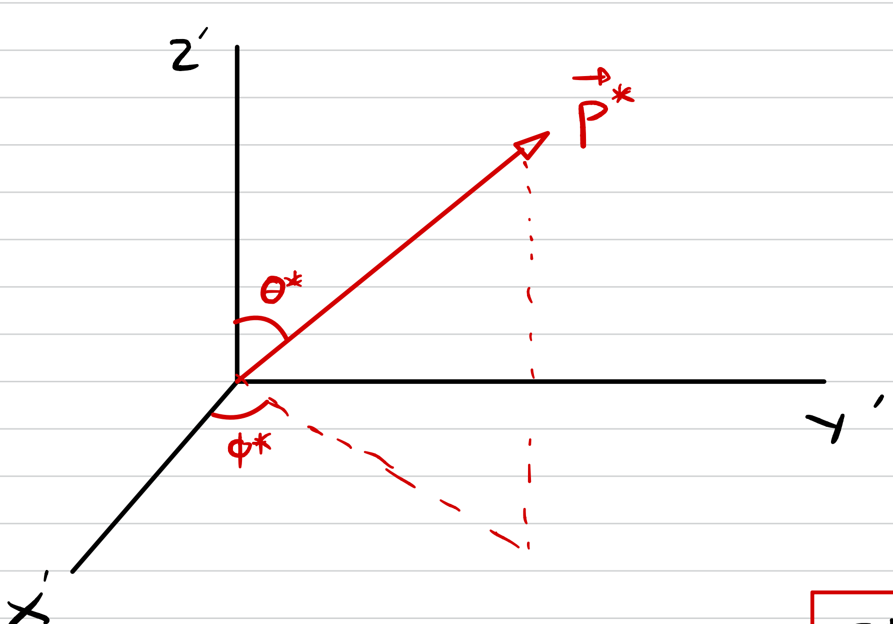
αναλλοιωτα Lorentz

βαθμοι ελευθεριας στο K.O. για δεδομενα M, m_1, m_2 ?

$P' = p_1' + p_2' \longrightarrow$

$3 \times 4 = 12$ παραμετροι
-4 εξισωσεις (δεσμοι)
-4 P^μ γνωστο στο K.O.
-2 $p_1^2 = m_1^2$ $p_2^2 = m_2^2$ $(+ - - -)$

2 βαθμοι ελευθεριας



θ^*, ϕ^*

$P' = (M, \vec{0})$
$p_1' = (E_1^*, \vec{p}^*)$
$p_2' = (E_2^*, -\vec{p}^*)$

$\vec{p}^* = (p^* \sin\theta^* \cos\phi^*, p^* \sin\theta^* \sin\phi^*, p^* \cos\theta^*)$

$$\boxed{E_1^* = \frac{M^2 + m_1^2 - m_2^2}{2M}, \quad E_2^* = \frac{M^2 + m_2^2 - m_1^2}{2M}}$$

$$\boxed{|\vec{P}_{1,2}^*|^2 = \frac{[M^2 - (m_1 - m_2)^2][M^2 - (m_1 + m_2)^2]}{4M^2}}$$

πληρως προσδιορισιμα συναρτησει των μαζων

$$\theta^* = \phi^* = 0$$

$$P' = (M, 0, 0, 0)$$

$$P_1' = (E_1^*, 0, 0, p^*)$$

$$P_2' = (E_2^*, 0, 0, -p^*)$$

$$P' = P_1' + P_2' \rightsquigarrow (P' - P_1')^2 = (P_2')^2$$

$(+ - - -)$

$$(P')^2 + (P_1')^2 - 2P' \cdot P_1' = m_2^2$$

$$M^2 + m_1^2 - 2ME_1^* = m_2^2$$

$$E_1^* = \frac{M^2 + m_1^2 - m_2^2}{2M}$$

ομοίως $E_2^* = \dfrac{M^2 + m_2^2 - m_1^2}{2M}$

$$\vec{P}_1^* \equiv \vec{P}^*$$

$$p \equiv |\vec{p}|$$

$$(E_1^*)^2 = m_1^2 + |\vec{P}_1^*|^2 = m_1^2 + (p^*)^2$$

$$(p^*)^2 = (E_1^*)^2 - m_1^2 = (E_2^*)^2 - m_2^2$$

$A = m_1 - m_2$ $\quad B = m_1 + m_2$ $\quad A^2 + B^2 = (m_1^2 + m_2^2)\, 2$

$$4M^2 (p^*)^2 = \left(M^2 + (m_1 - m_2)(m_1 + m_2)\right)^2 - m_1^2 4M^2 \qquad (1)$$

$$= \left(M^2 - (m_1 - m_2)(m_1 + m_2)\right)^2 - m_2^2 4M^2 \qquad (2)$$

$$(1) \rightarrow \quad = M^4 + (A \cdot B)^2 + 2A \cdot B M^2 - m_1^2 4M^2$$

$$(2) \rightarrow \quad = M^4 + (A \cdot B)^2 - 2A \cdot B M^2 - m_2^2 4M^2$$

$$\tfrac{1}{2}((1)+(2)) \quad = M^4 + (A \cdot B)^2 - 2(m_1^2 + m_2^2) M^2$$

$$= M^4 + (A \cdot B)^2 - (A^2 + B^2) M^2$$

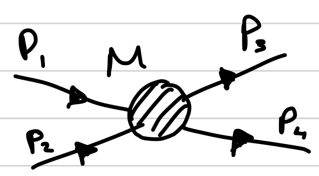
$$= M^2 (M^2 - A^2) - B^2 (M^2 - A^2)$$

$$4M^2 (p^*)^2 = (M^2 - B^2)(M^2 - A^2)$$

$$(p^*)^2 = |\vec{P}_{1,2}^*|^2 = \frac{[M^2 - (m_1 - m_2)^2]\,[M^2 - (m_1 + m_2)^2]}{4M^2}$$

2 → 2

$[m_1 + m_2 \xrightarrow{M} m_3 + m_4]$



σκεφτομαστε το 2→2 σαν 2→1 και 1→2 με ενδιαμεση "κατασταση" $(M, \vec{P})$

$P = (M, \vec{P})$ για $\vec{P} = 0$: κεντρο μαζας (μηδενικη ολικη ορμη)

$$p_1 + p_2 = p_3 + p_4$$

$$\boxed{E_1^* = \frac{M^2 + m_1^2 - m_2^2}{2M}, \quad E_2^* = \frac{M^2 + m_2^2 - m_1^2}{2M}} \;*$$

$$\boxed{|\vec{p}^*_{1,2}|^2 = \frac{[M^2 - (m_1 - m_2)^2]\,[M^2 - (m_1 + m_2)^2]}{4M^2}} \;*$$

για την τελικη κατασταση (p_3, p_4) αντικαθιστουμε $(1,2) \to (3,4)$

π.χ.

$$E_3^* = \frac{M^2 + m_3^2 - m_4^2}{2M}$$

$$|\vec{P}^*_{3,4}| = \frac{[M^2 - (m_3 - m_4)^2]\,[M^2 - (m_3 + m_4)^2]}{2M}$$

2 → 1

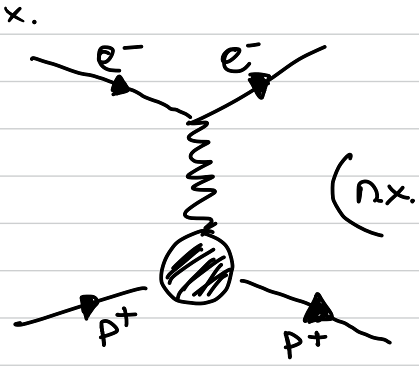
ομοιως με την 1 → 2

ελαστικη κρουση

Νευτωνια: $T_{πριν} = T_{μετα}$ {θερμοτητα, ηχος, τριβη}

Σχετικιστικα: $m_1\, m_2 \dots m_n \to m_1\, m_2 \dots m_n$ {ιδιες μαζες πριν + μετα}

π.χ.



(π.χ. $e^- p^+ \to e^- p^+$)

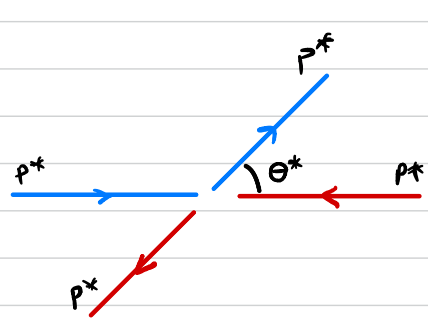
S^*

$E_1^* = E_3^*$ $\quad$ $E_2^* = E_4^*$

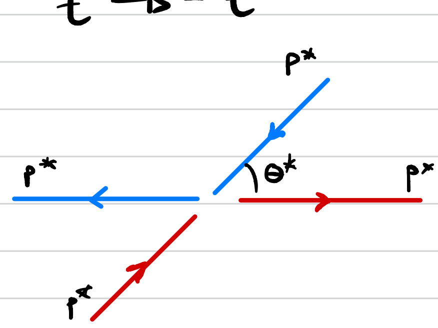
$|\vec{p}^*_{1,2}| = |\vec{p}^*_{3,4}| = |\vec{p}^*|$

$m_1 = m_3$ και $m_2 = m_4$

CM

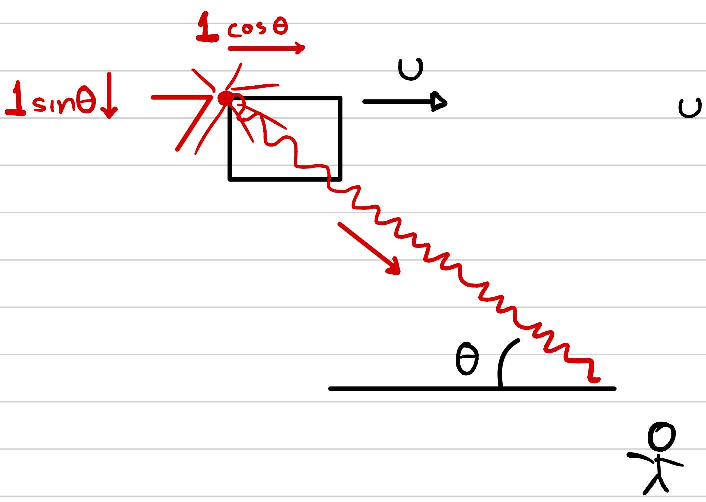


$t \to -t'$



συμμετρια αντιστροφης χρονου απαιτει οτι $|\vec{p}_1^*| = |\vec{p}_2^*| = |\vec{p}_3^*| = |\vec{p}_4^*| = p^*$!

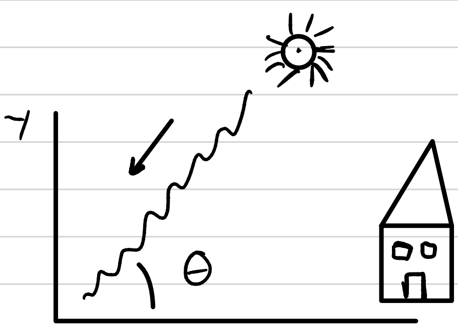
οπτικα φαινομενα

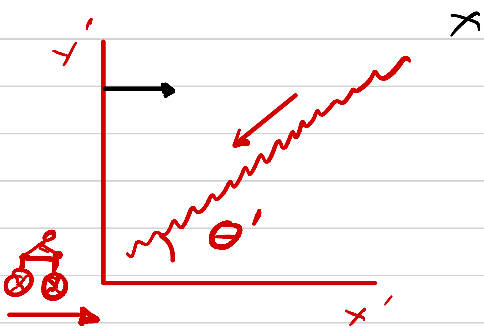


$\upsilon > \cos\theta$

μπορουμε να "δουμε" την συστολη Lorentz ενος αντικειμενου ?

αποπλανηση φωτος (aberration)





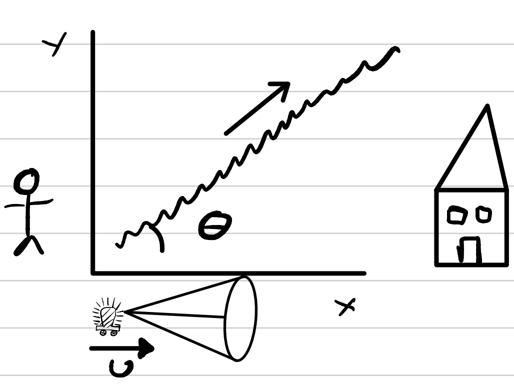
$$\begin{bmatrix}\gamma & -\gamma\upsilon & 0\\ -\gamma\upsilon & \gamma & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\varepsilon\\ -\varepsilon\cos\theta\\ -\varepsilon\sin\theta\end{bmatrix} = \varepsilon\begin{bmatrix}\gamma(1+\upsilon\cos\theta)\\ -\gamma(\upsilon+\cos\theta)\\ -\sin\theta\end{bmatrix} = \begin{bmatrix}\varepsilon'\\ -\varepsilon'\cos\theta'\\ -\varepsilon'\sin\theta'\end{bmatrix}$$

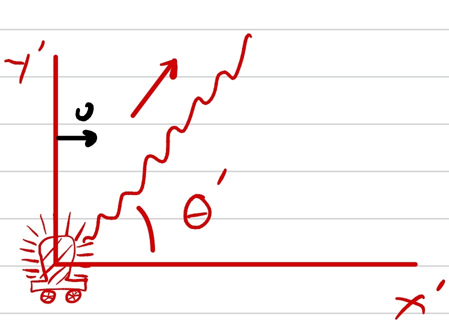
$$\cos\theta' = \frac{\upsilon+\cos\theta}{1+\upsilon\cos\theta} \qquad \sin\theta' = \frac{\sin\theta}{1+\upsilon\cos\theta}$$

$$\tan\left(\frac{\theta'}{2}\right) = \frac{\sin\theta'}{1+\cos\theta'} = \sqrt{\frac{1-\upsilon}{1+\upsilon}}\,\tan\left(\frac{\theta}{2}\right)$$

$\tan\frac{x}{2}$ = γνησιως μονοτονη για $x \in [0,\pi]$

αρα $\theta' < \theta$ για εισερχομενη ακτινα





φαινομενο προβολεα
headlight effect

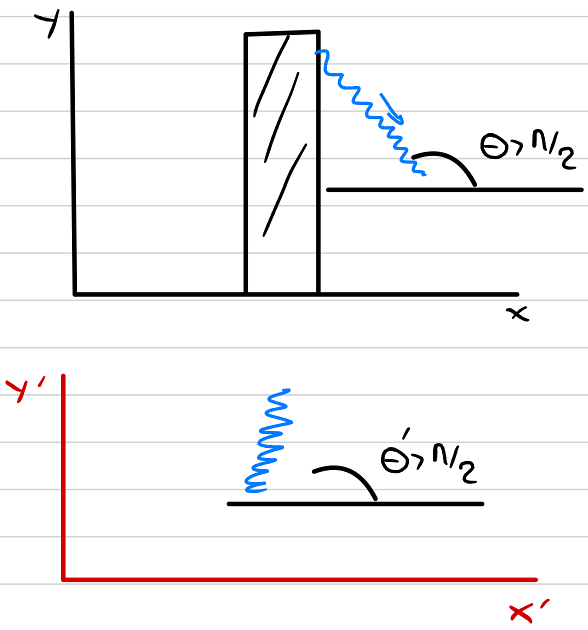
$t \to W^+ b \to \mu^+ \nu$ $(B^- \; K^\pm, \pi^\pm, \pi^0, \pi^+ \ldots)$

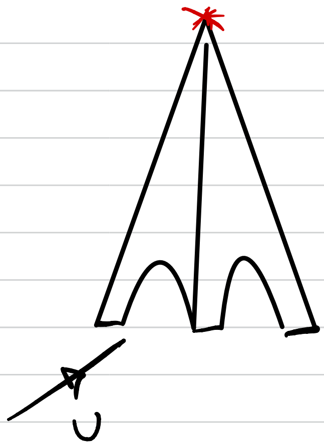
αδρονικο jet

boosted top quark tagging

LHC

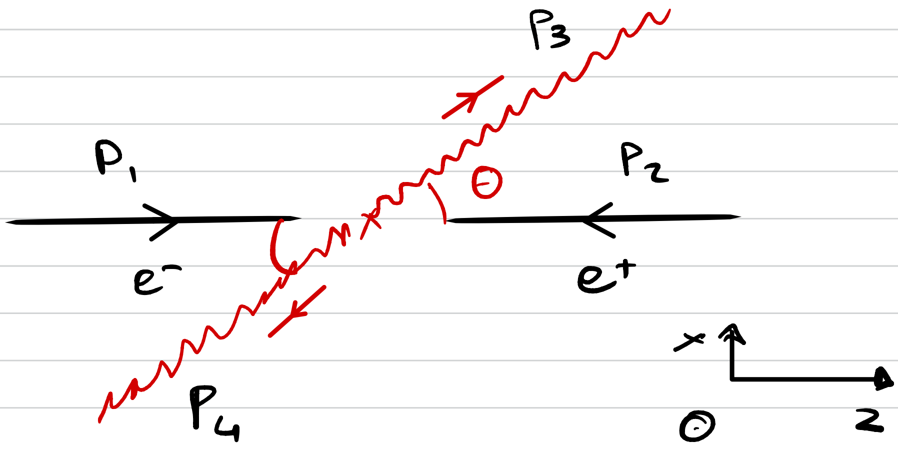
Θ. Αποστολάτος
04/06/20
Youtube





Youtube Video : Relativistic flight through the Eiffel Tower
Thomas Müller 2019

Εξαΰλωση $e^-e^- \to 2\gamma$



$m_{e^+} + m_{e^-} \longrightarrow 0 + 0$!

η μάζα των επιμέρους συστατικών δεν διατηρείτε πριν // μετά

η αναλλοίωτη μάζα του συστήματος ?

$E_\gamma^{min} = ?$

$$p_1 = \left(\sqrt{m_e^2+p^2},\, 0,\, 0,\, p\right) \qquad p_3 = \left(E_\gamma,\, 0,\, E_\gamma \sin\theta,\, E_\gamma \cos\theta\right)$$

$$p_2 = \left(\sqrt{m_e^2+p^2},\, 0,\, 0,\, -p\right) \qquad p_4 = \left(E_\gamma,\, 0,\, -E_\gamma \sin\theta,\, -E_\gamma \cos\theta\right)$$

$$p = E_\gamma \cos\theta$$
$$E_\gamma = \sqrt{m_e^2+p^2}$$

$E_\gamma^{min} = m_e = 511 \text{ keV}$

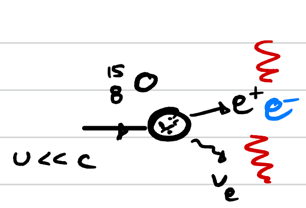
για $p \to 0$

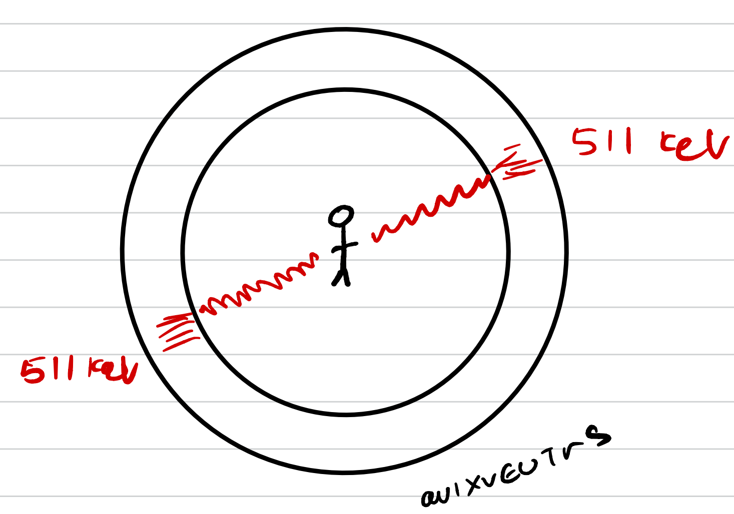
Positron Emission Tomography

ραδιοφάρμακο

$^{15}_{8}O \longrightarrow {}^{15}_{7}N\ e^+ \nu$

$p \to n\, e^+ \nu$

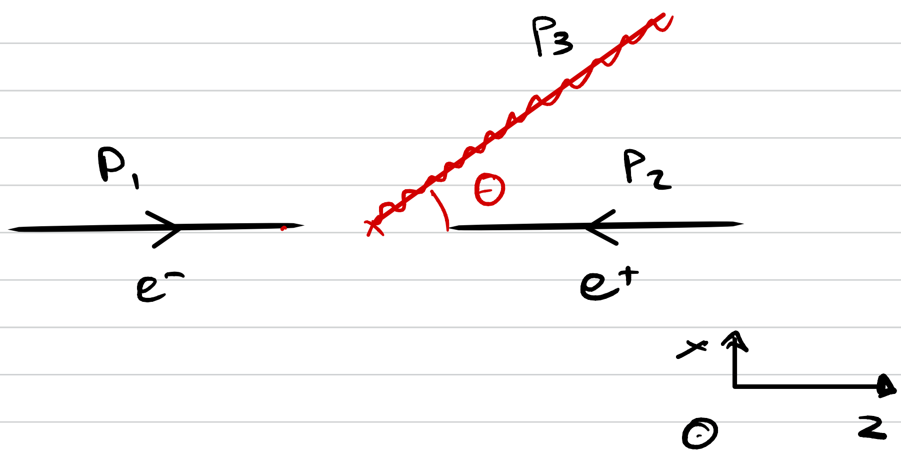




$\Delta\phi \simeq \pi$

$\Delta t \leq 0.5$ ns

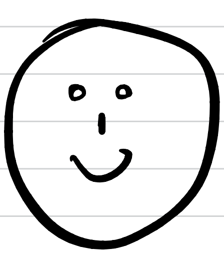
Εξαΰλωση $e^- e^+ \to \gamma$



$$E_\gamma^{min} = ?$$

$$p_1 = \left(\sqrt{m_e^2+p^2},\ 0,\ 0,\ p\right) \qquad p_3 = \left(E_\gamma,\ 0,\ E_\gamma \sin\theta,\ E_\gamma \cos\theta\right)$$

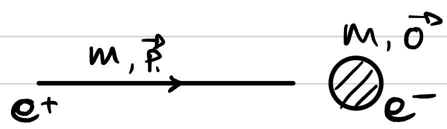
$$p_2 = \left(\sqrt{m_e^2+p^2},\ 0,\ 0,\ -p\right) \qquad p_4 = \left(0,\ 0,\ 0,\ 0\right)$$



$$p_1 + p_2 \overset{?}{=} p_3 + p_4$$

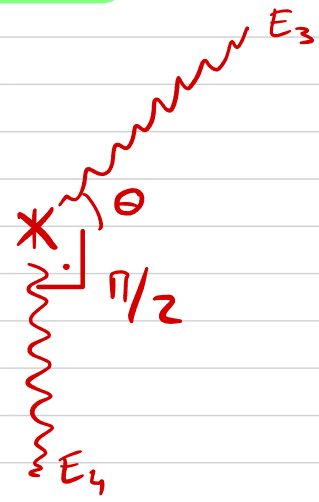
Προβλημα 8.4 (Taylor & Wheeler)

$T = m$



πριν

μετα



$E_3 = ?$

$E_4 = ?$

$\theta = ?$

$\boxed{S}$

$$P_1^\mu = \begin{pmatrix} \sqrt{m^2+p_1^2} \\ 0 \\ 0 \\ p_1 \end{pmatrix} \quad P_2^\mu = \begin{pmatrix} m \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad P_3^\mu = \begin{pmatrix} E_3 \\ E_3 \sin\theta \\ 0 \\ E_3\cos\theta \end{pmatrix} \quad P_4^\mu = \begin{pmatrix} E_4 \\ -E_4 \\ 0 \\ 0 \end{pmatrix}$$

$$\sqrt{m^2+p_1^2} + m = E_3 + E_4$$
$$E_3 \sin\theta = E_4$$
$$E_3 \cos\theta = p_1$$
$$T = \sqrt{m^2+p_1^2} - m = m$$

αγνωστα: E_3, E_4, p_1, θ

4 εξισωσεις

επιλυσιμο συστημα

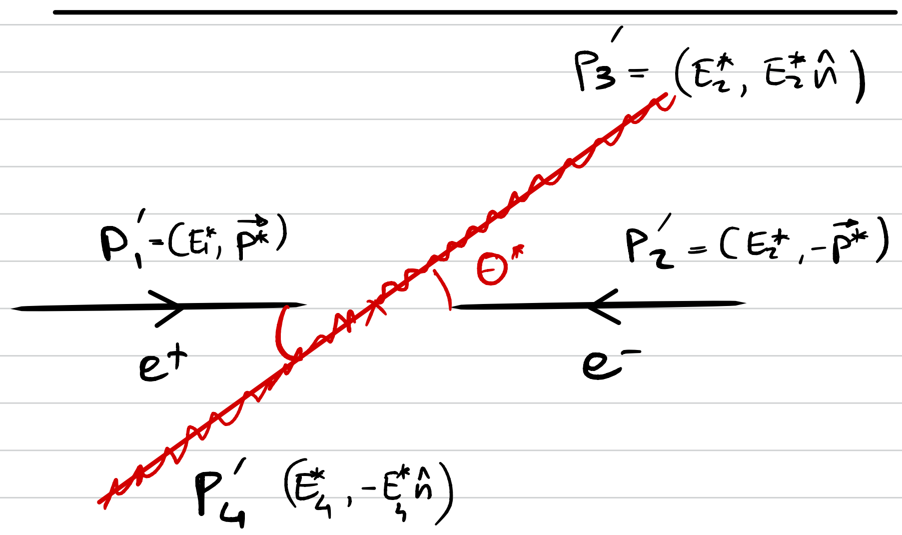
... ***

$\boxed{T \neq p^2/2m\,!}$

$$s = 2m^2 + 2m\left(\sqrt{m^2+p_1^2}\right) = 2m^2 + 2m(2m) = 6m^2$$

Στο $\boxed{S'}$ Κ.Ο.

$$\sqrt{s} = \sqrt{6}\, m = 2E_\gamma$$



Λογω συμμετριας

$$\left.\begin{aligned} E_1^* = E_2^* &= \sqrt{s}/2 \\ E_3^* = E_4^* &= \sqrt{s}/2 \end{aligned}\right\} = E^* = \sqrt{\tfrac{3}{2}}\, m$$

$$p^* = \sqrt{(E^*)^2 - m^2} = \frac{\sqrt{2}}{2} m$$

$$P_1^{\mu'} = \begin{bmatrix} E^* \\ 0 \\ 0 \\ p^* \end{bmatrix} \quad P_2^{\mu'} = \begin{bmatrix} E^* \\ 0 \\ 0 \\ -p^* \end{bmatrix} \quad P_3^{\mu'} = \begin{bmatrix} E^* \\ E^*\sin\theta^* \\ 0 \\ E^*\cos\theta^* \end{bmatrix} \quad P_4^{\mu'} = \begin{bmatrix} E^* \\ -E^*\sin\theta^* \\ 0 \\ -E^*\cos\theta^* \end{bmatrix}$$

1 αγνωστος στο Κ.Ο. $= \theta^*$ $\quad (\phi^* = 0 \rightsquigarrow$ επιπεδο xz$)$

$$\underset{\boxed{S}}{P_2} = \Lambda(-\vec{v}_{cm})\, \underset{\boxed{S'}}{P_2'}$$

$$\vec{U}_{cm} = -\vec{U}_2' = \frac{\vec{P}^*}{E^*} = \frac{\frac{\sqrt{2}}{2}m}{\sqrt{\frac{3}{2}}\,m}\,\hat{z} = \frac{1}{\sqrt{3}}\,\hat{z}$$

↳ ταχοτητα του S' ως προς το S

$$\gamma_{cm} = \frac{1}{\sqrt{(1-1/3)}} = \sqrt{3/2} = \frac{E^*}{m}$$

$$P^\mu_{cm} = \left(\gamma_{cm}M \,,\, \gamma_{cm} M \vec{U}_{cm}\right)$$

$$e^+e^- \to M \to \gamma\gamma$$

οπου $M = \sqrt{s}$

$$\boxed{p^* = \frac{\sqrt{2}}{2}m} \qquad \boxed{E^* = \sqrt{3/2}\; m}$$

Επιβεβαιωση

$$\underset{P_2^\mu}{\begin{pmatrix} m \\ 0 \\ 0 \\ 0 \end{pmatrix}} = \underset{\Lambda^\mu{}_{\mu'}(-\vec{U}_{cm})}{\begin{bmatrix} \sqrt{3/2} & 0 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & 0 & \sqrt{3/2} \end{bmatrix}} \underset{P_2^{\mu'}}{\begin{pmatrix} E^* \\ 0 \\ 0 \\ -p^* \end{pmatrix}} = \begin{pmatrix} \sqrt{\frac{3}{2}}E^* - \frac{\sqrt{2}}{2}p^* \\ 0 \\ 0 \\ \frac{\sqrt{2}}{2}E^* - \sqrt{\frac{3}{2}}\,p^* \end{pmatrix} = \text{OK}$$

$$\underset{P_4^\mu}{\begin{pmatrix} E_4 \\ -E_4 \\ 0 \\ 0 \end{pmatrix}} = \underset{\Lambda^\mu{}_{\mu'}}{\begin{bmatrix} \sqrt{3/2} & 0 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & 0 & \sqrt{3/2} \end{bmatrix}} \underset{P_4^{\mu'}}{\begin{bmatrix} E^* \\ -E^*\sin\theta^* \\ 0 \\ -E^*\cos\theta^* \end{bmatrix}} = \begin{bmatrix} \sqrt{\frac{3}{2}}\sqrt{\frac{3}{2}}m - \frac{\sqrt{2}}{2}\sqrt{\frac{3}{2}}\,m\cos\theta^* \\ -\sqrt{\frac{3}{2}}\,m\sin\theta^* \\ 0 \\ \frac{\sqrt{2}}{2}\sqrt{\frac{3}{2}}\,m - \sqrt{\frac{3}{2}}\sqrt{\frac{3}{2}}\,m\cos\theta^* \end{bmatrix}$$

$$\sqrt{\frac{3}{2}}\cos\theta^* = \frac{\sqrt{2}}{2} \Rightarrow \cos\theta^* = \frac{1}{\sqrt{3}} \qquad \sin\theta^* = \sqrt{\frac{2}{3}}$$

$$E_4 = \sqrt{\frac{3}{2}}\,m\,\sqrt{\frac{2}{3}} = m \qquad \text{αρα} \quad P_4^\mu = \begin{bmatrix} m \\ -m \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{3/2} & 0 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & 0 & \sqrt{3/2} \end{bmatrix} \begin{bmatrix} E^* \\ E^*\sin\theta^* \\ 0 \\ E^*\cos\theta^* \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}\sqrt{\frac{3}{2}}\,m + \frac{\sqrt{2}}{2}\sqrt{\frac{3}{2}}\frac{1}{\sqrt{3}}\,m \\ \sqrt{\frac{3}{2}}\,m \cdot \sqrt{\frac{2}{3}} \\ 0 \\ \frac{\sqrt{2}}{2}\sqrt{\frac{3}{2}}\,m + \sqrt{\frac{3}{2}}\sqrt{\frac{3}{2}}\,m\frac{1}{\sqrt{3}} \end{bmatrix} = \begin{bmatrix} 2m \\ m \\ 0 \\ \sqrt{3}\,m \end{bmatrix} = P_3^\mu$$

$$\tan\theta = \frac{1}{\sqrt{3}} \qquad \theta = \pi/6$$

$$\sqrt{m^2+p_1^2} + m = E_3 + E_4$$

$$E_3 \sin\theta = E_4$$

$$E_3 \cos\theta = p_1$$

$$T = \sqrt{m^2+p_1^2} - m = m$$

αγνωστα: E_3, E_4, p_1, θ

4 εξισωσεις

επιλυσιμο συστημα

...

$$\sqrt{m^2+p_1^2} = 2m \quad \Big| \quad m^2+p_1^2 = 4m^2 \quad \Big| \quad p_1 = \sqrt{3}\, m$$

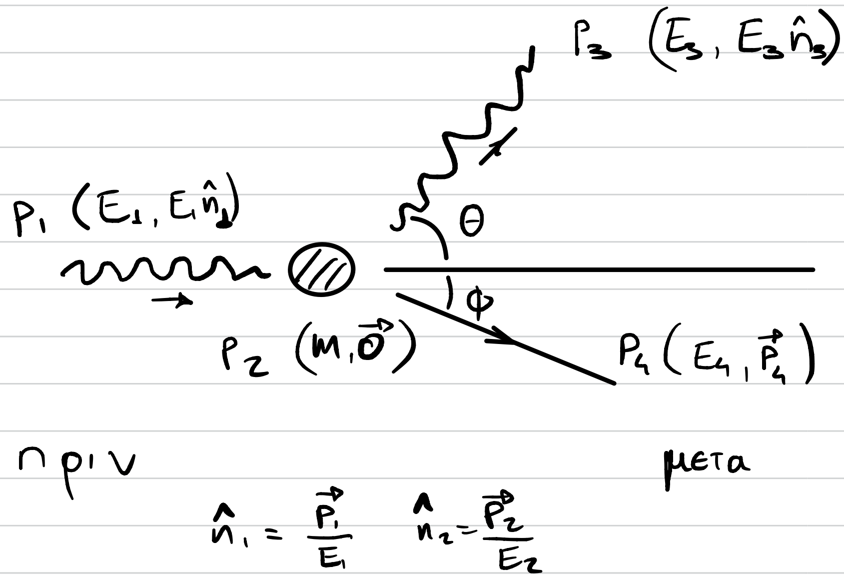
$$\left\{ \begin{array}{l} 3m = E_3 + E_4 \\ E_3 \sin\theta = E_4 \\ E_3\cos\theta = \sqrt{3}\, m \end{array} \right. \qquad \begin{array}{l} \rightarrow \quad E_3 = 3m - E_4 \\ \\ \Big\} \rightarrow \quad E_3^2 = E_4^2 + 3m^2 \end{array}$$

$$9m^2 + E_4^2 - 6mE_4 = E_4^2 + 3m^2$$

$$6m E_4 = 6m^2 \quad \rightarrow \quad \boxed{\begin{array}{l} E_4 = m \\ E_3 = 2m \\ \sin\theta = 1/2 \qquad \cos\theta = \frac{\sqrt{3}}{2} \end{array}}$$

***

Τι χρωμα εχει ενα πρωτονιο ;



$$p_1 + p_2 = p_3 + p_4$$
$$p_1 + p_2 - p_3 = p_4 \qquad (+\,-\,-\,-)$$
$$2p_1p_2 - 2p_1p_3 - 2p_2p_3 + m^2 = m^2$$
$$E_1 m - mE_3 = E_1E_3(1-\cos\theta)$$
$$E_1 - E_3 = \frac{E_1E_3}{m}(1-\cos\theta)$$
$$E_1 = \left[1 + \frac{E_1}{m}(1-\cos\theta)\right]E_3$$

$$\frac{1}{E_3} = \frac{1 + \frac{E_1}{m}(1-\cos\theta)}{E_1} = \frac{1}{E_1} + \frac{1}{m}(1-\cos\theta)$$

$$E = h\nu = h\frac{c}{\lambda}$$
$$\lambda = \frac{hc}{E}$$

$$\lambda_3 = \lambda_1 + \frac{hc}{m}(1-\cos\theta)$$

$\lambda = \frac{hc}{m}$ μηκος κυματος Compton σωματιδιου μαζας m

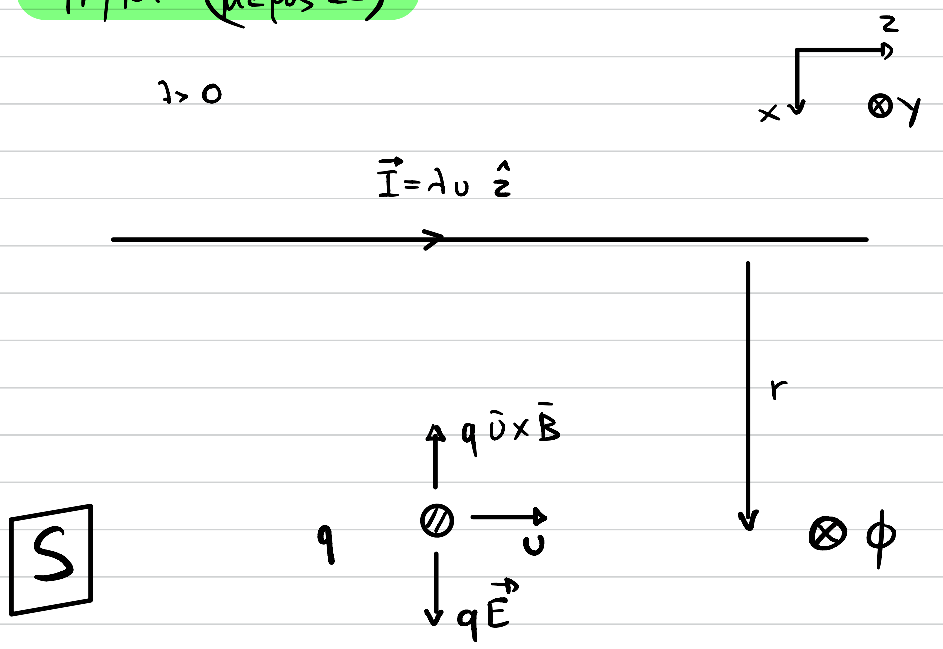
$\lambda_3 = \lambda_1 + \lambda$ για $\theta = \pm\frac{\pi}{2}$

$m = hc/\lambda$ μηκος κυματος που θα ειχε ενα φωτονιο ενεργειας $E = m$

Απ: το "χρωμα" ενος πρωτονιου αντιστοιχει σε μηκος κυματος $\lambda \simeq 1.3$ fm

$$p_1 - p_3 = p_4 - p_2 \qquad \text{εναλλακτικα}$$
$$-2p_1p_3 = -2p_4p_2 + 2m^2$$
$$E_1E_3(1-\cos\theta) = E_4 m - m^2$$
$$E_1E_3(1-\cos\theta) = m(E_1 + m - E_3) - m^2$$
$$E_1E_3(1-\cos\theta) = m(E_1 - E_3)$$

Η/Μ (μέρος 1ο)



$$\oint \vec{B}\, d\vec{\ell} = \mu_0 I$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi} \quad \Big|\ \text{για } \phi = 0$$

$$\bar{\upsilon}\times\bar{B} = (\upsilon\,\hat{z})\times\left(\frac{\mu_0 I}{2\pi r}\hat{\phi}\right) = \frac{\mu_0 I \upsilon}{2\pi r}(-\hat{r})$$

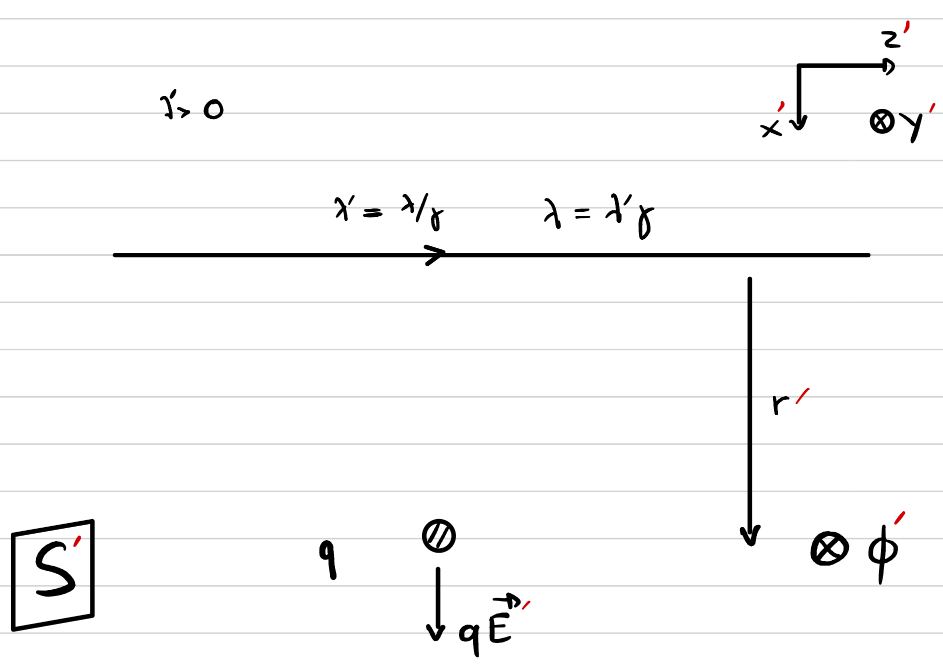
$$\oiint \vec{E}\, d\vec{S} = q/\varepsilon_0 \quad \Big|\ \vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$

$$\hat{r}\,(2\pi r L)\, E\,\hat{r} = \frac{L\cdot\lambda}{\varepsilon_0}$$

$$\vec{F}_L = q(\vec{E} + \vec{\upsilon}\times\vec{B})$$

$$\vec{F}_L = q\left(\frac{\lambda}{2\pi\varepsilon_0 r} - \frac{\mu_0\lambda\upsilon^2}{2\pi r}\right)\hat{r} = \frac{q\lambda}{2\pi\varepsilon_0 r}(1 - \mu_0\varepsilon_0\upsilon^2) = \frac{q\lambda}{2\pi\varepsilon_0 r}\left(1 - \frac{\upsilon^2}{c^2}\right)$$

$$\mu_0\varepsilon_0 = \frac{1}{c^2} \qquad \gamma = \frac{1}{\sqrt{1-\upsilon^2/c^2}} \quad \Big|\ \vec{F}_L = \frac{q\lambda/\gamma^2}{2\pi\varepsilon_0 r}\hat{r}$$



$$\vec{F}_L' = q\frac{\lambda'}{2\pi\varepsilon_0 r}\hat{r}$$

$$\vec{F}_L = \frac{q\lambda'/\gamma}{2\pi\varepsilon_0 r}\hat{r}$$

$$\boxed{\vec{F}_L = \vec{F}_L'/\gamma}$$

$$S' = \Lambda(\upsilon\hat{z})\, S$$

$$\begin{cases} x' = x - Vt \\ t' = t' \\ \upsilon' = \upsilon - V \\ \vec{a}' = \vec{a} \end{cases}$$

Γαλιλαίου

$$\vec{F} = m\vec{a}$$

$$\vec{E}' = \frac{\lambda/\gamma}{2\pi\varepsilon_0 r}\hat{r} \qquad \boxed{S'}$$
$$\vec{B}' = \vec{0}$$

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} \qquad \boxed{S}$$
$$\vec{B} = \frac{\mu_0\lambda\upsilon}{2\pi r}\hat{\phi}$$

$$\bar{E}'_{\parallel} = \bar{E}_{\parallel} \qquad \bar{E}'_T = \gamma(\bar{E}_T + \bar{\upsilon}\times\bar{B}_T)$$

$$\bar{B}'_{\parallel} = \bar{B}_{\parallel} \qquad \bar{B}'_T = \gamma(\bar{B}_T - \bar{\upsilon}\times\bar{E}_T)$$

$\bar{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ $\qquad \nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$

Εξισώσεις Maxwell

$$\bar{\nabla}\cdot\bar{E} = \rho/\varepsilon_0 \qquad (1)$$
$$\bar{\nabla}\cdot\bar{B} = 0 \qquad (2)$$
$$\bar{\nabla}\times\bar{E} + \frac{\partial \bar{B}}{\partial t} = 0 \qquad (3)$$
$$\bar{\nabla}\times\bar{B} - \mu_0\varepsilon_0\frac{\partial \bar{E}}{\partial t} = \mu_0\bar{J} \qquad (4)$$

$$\underbrace{\bar{\nabla}(\cancel{\bar{\nabla}}\times\bar{B})}_{0} - \mu_0\varepsilon_0\frac{\partial(\bar{\nabla}\bar{E})}{\partial t} = \mu_0\vec{\nabla}\vec{J}$$
$$\varepsilon_0\frac{\partial}{\partial t}(\rho/\varepsilon_0) + \vec{\nabla}\vec{J} = 0 \quad \Big| \quad \frac{\partial\rho}{\partial t} + \vec{\nabla}\vec{J} = 0$$

στο κενο $\rho = 0 \quad \vec{J} = 0$ $\quad \Big| \quad \bar{A}\times(\bar{B}\times\bar{C}) = \bar{B}(\bar{A}\bar{C}) - \bar{C}(\bar{A}\bar{B})$

$\bar{\nabla}\times(4)$

$$\vec{F}_L = q(\vec{E} + \vec{u}\times\vec{B})$$

$$\bar{\nabla}\times(\bar{\nabla}\times\bar{B}) = \bar{\nabla}(\cancelto{0}{\bar{\nabla}\cdot\bar{B}}) - \nabla^2\bar{B}$$
$$-\mu_0\varepsilon_0\partial_t(\bar{\nabla}\times\bar{E}) = -\mu_0\varepsilon_0\partial_t\left(-\frac{\partial\bar{B}}{\partial t}\right) = \mu_0\varepsilon_0\frac{\partial^2\bar{B}}{\partial t^2}$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$\frac{\partial^2\bar{B}}{\partial t^2} - \frac{1}{\mu_0\varepsilon_0}\nabla^2\bar{B} = 0 \quad \Big| \quad \nabla^2\bar{E} - \frac{1}{c^2}\frac{\partial^2\bar{E}}{\partial t^2} = 0$$

$$\nabla^2\bar{B} - \frac{1}{c^2}\frac{\partial^2\bar{B}}{\partial t^2} = 0$$

$$\vec{E} = \vec{E}_0\cos(\vec{k}\vec{x} - \omega t)$$

$E = \hbar\omega$

$\vec{p} = \vec{k}\hbar$

$\hbar = h/2\pi$

$$\phi(\vec{k},\omega,\vec{x},t) = \phi(E,\vec{p},\vec{x},\vec{t})$$
$$\vec{k}\vec{x} - \omega t = \frac{1}{\hbar}(\vec{p}\cdot\vec{x} - \vec{E}\cdot t) = \frac{1}{\hbar}p_\mu\cdot x^\mu = \frac{1}{\hbar}p^\mu x_\mu$$
$$\phi'(x',t') = \phi(x,t)$$

$$\boxed{\begin{array}{ll} \bar{E}'_{\parallel} = \bar{E}_{\parallel} & \bar{E}'_T = \gamma(\bar{E}_T + \bar{u}\times\bar{B}_T) \\ \bar{B}'_{\parallel} = \bar{B}_{\parallel} & \bar{B}'_T = \gamma(\bar{B}_T - \bar{u}\times\bar{E}_T) \end{array}}$$

$$\Delta x'^{\mu} \equiv \Delta x^{\mu'} = \Lambda^{\mu'}{}_{\mu}\cdot\Delta x^{\mu}$$
$$p^{\mu'} = \Lambda^{\mu'}{}_{\mu}\,p^{\mu}$$
$$u^{\mu'} = \Lambda^{\mu'}{}_{\mu}\,u^{\mu}$$

$$\Lambda^{\mu'}{}_{\mu} = \begin{bmatrix} \gamma & -\gamma u & 0 & 0 \\ -\gamma u & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

contravariant
αντταλλοιωτα

$$\Delta x'_{\mu} \equiv \Delta x_{\mu'} = \Lambda_{\mu'}{}^{\mu}\Delta x_{\mu}$$

συναλλοιωτα (covariant)

$$\Lambda_{\mu'}{}^{\mu} = \begin{bmatrix} \gamma & \gamma u & 0 & 0 \\ \gamma u & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Τανυστης δυναμης Η/Μ $\quad F^{\mu\nu} \quad$ (rank 2)

$$F^{\mu'\nu'} = F'^{\mu\nu}(x') = \Lambda^{\mu}{}_{\kappa}\Lambda^{\nu}{}_{\lambda}F^{\kappa\lambda}(x) \rightarrow \Lambda F\cdot\Lambda^T$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix} \quad \Big| \quad F^{\mu\nu} = -F^{\nu\mu} \quad \Big| \quad C = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ i \end{bmatrix} \quad A = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ i \end{bmatrix}$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & E^T \\ E & B \end{bmatrix}$$

$$C_i = B_{ij}A_j = \sum_j B_{ij}A_j$$

$$C_i = B_{ik}A_k = \sum_k B_{ik}A_k$$

"βουβοι" δεικτες

$$F'^{\mu\nu} = \begin{bmatrix} \gamma & -\gamma u & 0 & 0 \\ -\gamma u & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix} \begin{bmatrix} \gamma & -\gamma u & 0 & 0 \\ -\gamma u & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\gamma^2 = \frac{1}{1-u^2}$$

$$\gamma^2(1-u^2) = 1$$

$$\begin{bmatrix} \gamma u E_x & \gamma E_x & \gamma(E_y - uB_z) & \gamma(E_z + uB_y) \\ -\gamma E_x & -\gamma(uE_x) & -\gamma(uE_y - B_z) & -\gamma(uE_z + B_y) \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix} \begin{bmatrix} \gamma & -\gamma u & 0 & 0 \\ -\gamma u & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & E_x & \gamma(E_y - uB_z) & \gamma(E_z + uB_y) \\ -E_x & 0 & -\gamma(uE_y - B_z) & -\gamma(uE_z + B_y) \\ -\gamma(E_y - uB_z) & \gamma(uE_y - B_z) & 0 & B_x \\ -\gamma(E_z + uB_y) & \gamma(uE_z + B_y) & -B_x & 0 \end{bmatrix} = F'^{\mu\nu}$$

$$\begin{bmatrix} 0 & E_x' & E_y' & E_z' \\ -E_x' & 0 & B_z' & -B_y' \\ -E_y' & -B_z' & 0 & B_x \\ -E_z' & B_y' & -B_x & 0 \end{bmatrix} = F'^{\mu\nu}$$

$$E_y' = \gamma(E_y - uB_z) \qquad E_z' = \gamma(E_z + uB_y)$$

$$\bar{E}_\perp' = \gamma(\bar{E}_T + \bar{u} \times \bar{B}_T) = \gamma(\bar{E}_T + (u\hat{x}) \times (B_y \hat{y} + B_z \hat{z}))$$

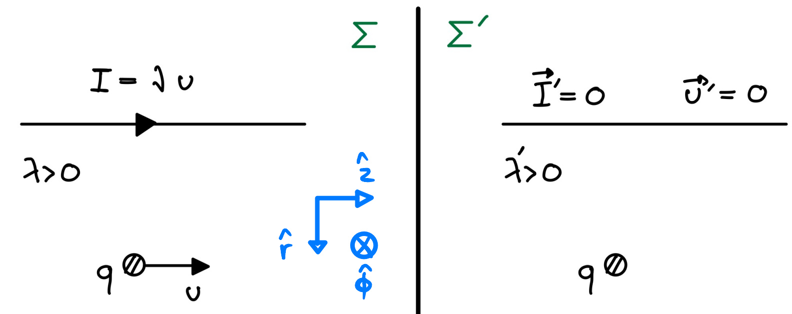
$$\bar{u} = u\hat{x} \qquad \begin{aligned} \hat{x} \times \hat{y} &= \hat{z} \\ \hat{x} \times \hat{z} &= -\hat{y} \end{aligned}$$

Η/Μ (μερος 2ο)

μετασχηματισμος Lorentz στα Η/Μ

$\vec{E},\vec{B} \longrightarrow \vec{E}',\vec{B}'$

π.χ.



$\vec{E} = \dfrac{\lambda}{2\pi\varepsilon_0 r}\,\hat{r}$ | $\vec{E}' = \dfrac{\lambda/\gamma}{2\pi\varepsilon_0 r}\,\hat{r}$

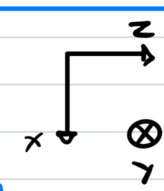
$\vec{B} = \dfrac{\mu_0 \lambda \upsilon}{2\pi r}\,\hat{\phi}$ | $\vec{B}' = \vec{0}$

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}$$

$$T'^{\mu\nu} \equiv \Lambda^{\mu}{}_{\kappa} \Lambda^{\nu}{}_{\lambda} T^{\kappa\lambda}$$

$$T'^{\mu} \equiv \Lambda^{\mu}{}_{\nu} T^{\nu}$$

$\phi = 0$



$\vec{E} = \dfrac{\lambda}{2\pi\varepsilon_0 r}\,\hat{x}$ (A) $\qquad \vec{E}' = \dfrac{\lambda/\gamma}{2\pi\varepsilon_0 r}\,\hat{x}$

$\vec{B} = \dfrac{\mu_0 \lambda \upsilon}{2\pi r}\,\hat{y} \qquad \vec{B}' = \vec{0}$

$$F^{\mu\nu} = \begin{bmatrix} 0 & A & 0 & 0 \\ -A & 0 & 0 & -A\upsilon \\ 0 & 0 & 0 & 0 \\ 0 & A\upsilon & 0 & 0 \end{bmatrix}$$

$\vec{E} = A\,\hat{x}$ | $\vec{E} = A\,\hat{x}$

$\vec{B} = \mu_0\varepsilon_0 A\upsilon\,\hat{y}$ | $\vec{B} = A\upsilon\,\hat{y}$

$c^2 = \dfrac{1}{\mu_0\varepsilon_0} \qquad c = 1$

$$F'^{\mu\nu} = \Lambda\, F^{\mu\nu}\, \Lambda^{T}$$

$$\gamma(1-\upsilon^2) = \frac{1}{\sqrt{1-\upsilon^2}}(1-\upsilon^2) = \frac{1}{\gamma}$$

$$F'^{\mu\nu} = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\upsilon \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\upsilon & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} 0 & A & 0 & 0 \\ -A & 0 & 0 & -A\upsilon \\ 0 & 0 & 0 & 0 \\ 0 & A\upsilon & 0 & 0 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\upsilon \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\upsilon & 0 & 0 & \gamma \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & A/\gamma & 0 & 0 \\ -A & 0 & 0 & -A\upsilon \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\upsilon \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\upsilon & 0 & 0 & \gamma \end{bmatrix} = \begin{bmatrix} 0 & A/\gamma & 0 & 0 \\ -A/\gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & E'_x & E'_y & E'_z \\ -E'_x & 0 & B'_z & -B'_y \\ -E'_y & -B'_z & 0 & B'_x \\ -E'_z & B'_y & -B'_x & 0 \end{bmatrix}$$

$\vec{B}' = 0 \qquad \vec{E}' = A/\gamma\;\hat{x}$

$$T'^{31} = \cancel{\Lambda^3{}_0}\Lambda^1{}_0 T^{00} + \cancel{\Lambda^3{}_0}\Lambda^1{}_1 T^{01} + \cancel{\Lambda^3{}_0}\Lambda^1{}_2 T^{02} + \cancel{\Lambda^3{}_0}\Lambda^1{}_3 T^{03}$$
$$+ \cancel{\Lambda^3{}_1}\Lambda^1{}_0 T^{10} + \cancel{\Lambda^3{}_1}\Lambda^1{}_1 T^{11} + \cancel{\Lambda^3{}_1}\Lambda^1{}_2 T^{12} + \cancel{\Lambda^3{}_1}\Lambda^1{}_3 T^{13}$$
$$+ \cancel{\Lambda^3{}_2}\Lambda^1{}_0 T^{20} + \cancel{\Lambda^3{}_2}\Lambda^1{}_1 T^{21} + \cancel{\Lambda^3{}_2}\Lambda^1{}_2 T^{22} + \cancel{\Lambda^3{}_2}\Lambda^1{}_3 T^{23}$$
$$+ \Lambda^3{}_3\Lambda^1{}_0 T^{30} + \Lambda^3{}_3\Lambda^1{}_1 T^{31} + \Lambda^3{}_3\cancel{\Lambda^1{}_2} T^{32} + \Lambda^3{}_3\cancel{\Lambda^1{}_3} T^{33}$$

γενικα

ειδικα για x

$$\Lambda^\mu{}_\nu = \begin{bmatrix} \gamma & -\gamma\upsilon & 0 & 0 \\ -\gamma\upsilon & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Lambda(\bar{\upsilon} = \upsilon\hat{x})$$

$$T'^{31} = \Lambda^3{}_3\Lambda^1{}_0 T^{30} + \Lambda^3{}_3\Lambda^1{}_1 T^{31} =$$

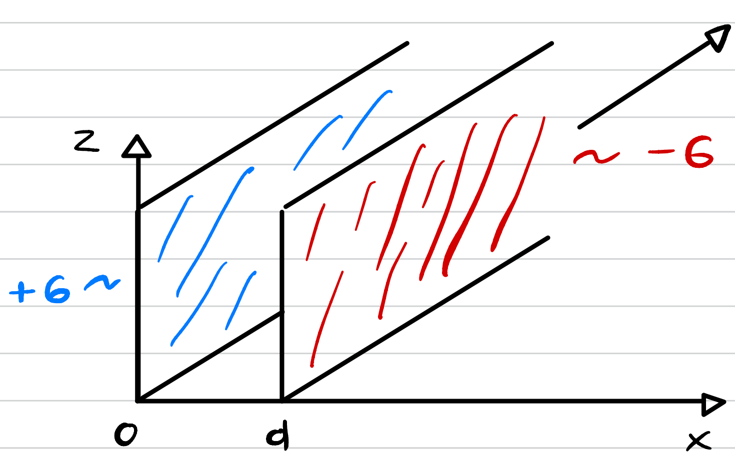
$$T'^{31} = \gamma\left(T^{31} - \upsilon T^{30}\right)$$

εαν $T^{\mu\nu} = F^{\mu\nu}$ $\longrightarrow$

$$\boxed{\begin{aligned} \bar{E}'_{\parallel} &= \bar{E}_{\parallel} & \bar{E}'_T &= \gamma(\bar{E}_T + \bar{\upsilon}\times\bar{B}_T) \\ \bar{B}'_{\parallel} &= \bar{B}_{\parallel} & \bar{B}'_T &= \gamma(\bar{B}_T - \bar{\upsilon}\times\bar{E}_T) \end{aligned}}$$

πυκνωτης

απειρες πλακες στο επιπεδο yz στις θεσεις $x=0$ και $x=d$



$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{x} \qquad 0<x<d$$

$$\vec{B} = \vec{0}$$

περιπτωσεις :

#1 $\Lambda(\bar{v} = v\hat{x})$

#2 $\Lambda(\bar{v} = v\hat{z})$

#1

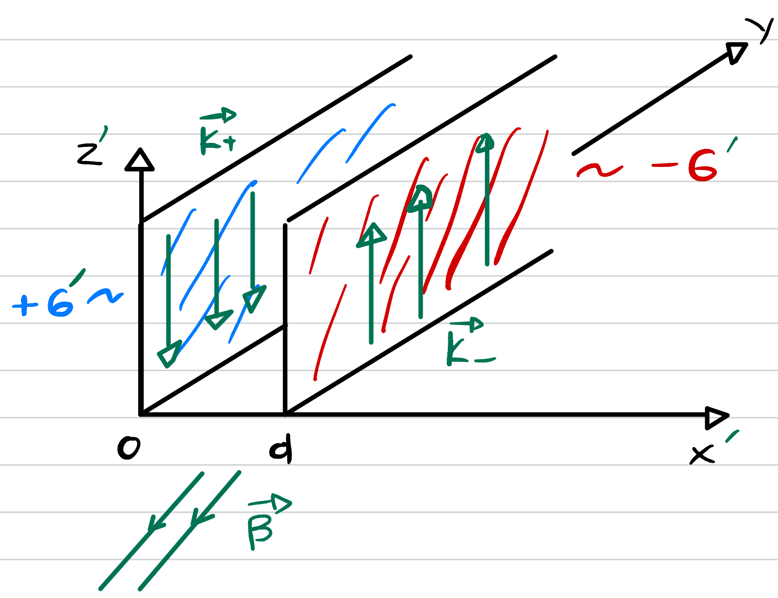
$$\vec{E}'_{\parallel} = \vec{E}_{\parallel} = \frac{\sigma}{\varepsilon_0}\hat{x} \qquad \bar{E}'_T = \gamma(\bar{0} + \bar{v}\times\bar{0})$$

$$\vec{B}'_{\parallel} = \vec{B}_{\parallel} = 0\,\hat{x} \qquad \bar{B}'_T = \gamma(\bar{0} - \bar{v}\times\bar{0})$$

$$\boxed{\begin{array}{ll} \bar{E}'_{\parallel} = \bar{E}_{\parallel} & \bar{E}'_T = \gamma(\bar{E}_T + \bar{v}\times\bar{B}_T) \\ \bar{B}'_{\parallel} = \bar{B}_{\parallel} & \bar{B}'_T = \gamma(\bar{B}_T - \bar{v}\times\bar{E}_T) \end{array}}$$

#2

$\Lambda(\bar{v} = v\hat{z})$



$$\vec{E}' = \frac{\sigma'}{\varepsilon_0}\hat{x} \qquad 0<x<d$$

$$\vec{B}' = -\mu_0\sigma' v\,\hat{y}$$

$$\sigma' = \sigma\cdot\gamma$$

$$\sigma/\varepsilon_0 \equiv A$$

$$\left.\begin{array}{l} \vec{E}' = (\gamma A)\hat{x} = E_x' = \hat{x} \\ \vec{B}' = -\mu_0(\sigma\cdot\gamma)\cdot v\,\hat{y} = -\mu_0\varepsilon_0 A\gamma v\,\hat{y} \underset{c=1}{=} -\gamma v A\,\hat{y} \end{array}\right\}$$

#2   #1

$$F'^{\mu\nu} = \begin{bmatrix} \gamma & 0 & 0 & -\gamma v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma v & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} 0 & A & 0 & 0 \\ -A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma v & 0 & 0 & \gamma \end{bmatrix} =$$

συμβατα

$$= \begin{bmatrix} 0 & \gamma A & 0 & 0 \\ -A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -\gamma v A & 0 & 0 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma v & 0 & 0 & \gamma \end{bmatrix} = \begin{bmatrix} 0 & \overset{E_x'}{\gamma A} & 0 & 0 \\ -\gamma A & 0 & 0 & \gamma v A \\ 0 & 0 & 0 & 0 \\ 0 & -\gamma v A & 0 & 0 \end{bmatrix} \sim -B_y$$

Εξίσωση κίνησης φορτίου εντός Η/Μ πεδίου

$$\frac{dp^\mu}{d\tau} = \frac{d}{d\tau}\begin{bmatrix} E \\ \vec{p} \end{bmatrix} = \begin{bmatrix} \gamma\, dE/dt \\ \gamma\, d\vec{p}/dt \end{bmatrix} = q\, F^{\mu\nu} \cdot U_\nu = q \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix} \begin{bmatrix} -\gamma \\ \gamma u_x \\ \gamma u_y \\ \gamma u_z \end{bmatrix}$$

$$= \begin{bmatrix} \gamma(E_x u_x + E_y u_y + E_z u_z) \\ \gamma(E_x + B_z u_y - B_y u_z) \\ \gamma(E_y - B_z u_x + B_x u_z) \\ \gamma(E_z + B_y u_x - B_x u_y) \end{bmatrix} = \begin{bmatrix} \gamma(q\vec{E}\cdot\vec{u}) \\ \gamma q(\vec{E} + \vec{u}\times\vec{B}) \end{bmatrix} = \begin{bmatrix} \gamma(\vec{F}_L\cdot\vec{u}) \\ \gamma \vec{F}_L \end{bmatrix}$$

$$\bar{u} \times \bar{B} = (\varepsilon_{ijk} u_j B_k)_i$$

$$\vec{u}\cdot\vec{F}_L = \vec{u}\, q(\vec{E} + \vec{u}\times\vec{B}) = q\,\vec{E}\cdot\vec{u}$$

$$(\bar{u}\times\bar{B})_x = u_y B_z - u_z B_y$$
$$(\bar{u}\times\bar{B})_y = B_x u_z - B_z u_x$$
$$(\bar{u}\times\bar{B})_z = B_y u_x - B_x u_y$$

$$\boxed{d\vec{p}/dt = \frac{d(\gamma m \vec{u})}{dt} = \vec{F}_L = q(\vec{E} + \vec{u}\times\vec{B})}$$

$$F^{\mu} = q \underbrace{\begin{bmatrix} 0 & A & 0 & 0 \\ -A & 0 & 0 & -Av \\ 0 & 0 & 0 & 0 \\ 0 & Av & 0 & 0 \end{bmatrix}}_{F^{\mu\nu}} \underbrace{\begin{bmatrix} -\gamma \\ 0 \\ 0 \\ \gamma v \end{bmatrix}}_{U_\nu} = \begin{bmatrix} 0 \\ qA\gamma(1-v^2) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ qA/\gamma \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \gamma \cdot \vec{F}_L \cdot \hat{x} \\ 0 \\ 0 \end{bmatrix} \qquad \vec{F}_L = q\frac{A}{\gamma^2}\hat{x}$$

$$F^{\mu'} = q \underbrace{\begin{bmatrix} 0 & A/\gamma & 0 & 0 \\ -A/\gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{F^{\mu\nu'}} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ qA/\gamma \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \cdot \vec{F}_L' \cdot \hat{x} \\ 0 \\ 0 \end{bmatrix} \qquad \vec{F}_L' = \frac{qA}{\gamma}\hat{x}$$

$$F^{\mu} = \begin{bmatrix} \gamma\, dE/dt \\ \gamma \vec{F} \end{bmatrix}$$

$$\vec{F}_L' = \vec{F}_L$$

$$\boxed{\vec{F}_L = \vec{F}_L' / \gamma}$$

Η/Μ (3ο μέρος)

Θεώρημα 5.1 (Χριστοδουλάκη & Κορφιάτη)

Αν μια εξίσωση τανυστών είναι αληθής σε ένα αδρανειακό σύστημα συντεταγμένων, τότε είναι αληθής και σε κάθε άλλο

Δύναμη Minkowski

$p^\mu = m u^\mu$, $\frac{dm}{d\tau} = 0$

$$\frac{dp^\mu}{d\tau} = F^\mu = m\frac{du^\mu}{d\tau} = m a^\mu = q F^{\mu\nu} u_\nu \qquad \Big| \qquad \vec{F}_L = q(\vec{E} + \vec{u}\times\vec{B})$$

$$0 = F^\mu \cdot u_\mu = q F^{\mu\nu} u_\nu u_\mu \overset{\text{πρέπει}}{=} 0 \qquad \Big| \qquad \frac{d(m u^\mu)}{d\tau} u_\mu = \frac{d}{d\tau}\left(\frac{1}{2} m u_\mu u^\mu\right) = 0$$

$$q F^{\mu\nu} u_\nu u_\mu = \frac{1}{2} F^{\mu\nu} u_\nu u_\mu + \frac{1}{2} F^{\nu\mu} u_\mu u_\nu = \frac{1}{2} u_\mu u_\nu \left(F^{\mu\nu} + F^{\nu\mu}\right) = 0$$

$F^{\mu\nu} = -F^{\nu\mu}$ αντισυμμετρικός $\qquad F^{\mu'\nu'} = \Lambda^{\mu'}{}_{\mu} \Lambda^{\nu'}{}_{\nu} F^{\mu\nu}$

$$\boxed{\begin{array}{ll} T^{\mu'\nu'} = \Lambda^{\mu'}{}_{\mu} \Lambda^{\nu'}{}_{\nu} T^{\mu\nu} & A^\mu = \eta^{\mu\nu} A_\nu \\ T^{\mu'\nu'}{}_{\kappa'} = \Lambda^{\mu'}{}_{\mu} \Lambda^{\nu'}{}_{\nu} \Lambda_{\kappa'}{}^{\kappa} T^{\mu\nu}{}_{\kappa} & A_\mu = \eta_{\mu\nu} A^\nu \end{array}}$$

polar

$$T^{\mu\nu} = \begin{bmatrix} 0 & T^{01} & T^{02} & T^{03} \\ T^{10} & 0 & T^{12} & T^{13} \\ T^{20} & T^{21} & 0 & T^{23} \\ T^{30} & T^{31} & T^{32} & 0 \end{bmatrix}$$

axial

μετασχηματίζονται σαν 3-vector σε στροφές // κατοπτρισμούς

$T^{00} = -T^{00} = 0$

$T^{01} = -T^{10}$

6 ανεξάρτητοι βαθμοί ελευθερίας

$\vec{E}, \vec{B}$

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix} \qquad \tilde{F}^{\mu\nu} = \begin{bmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z & E_y \\ -B_y & E_z & 0 & -E_x \\ -B_z & -E_y & E_x & 0 \end{bmatrix}$$

δυϊκός τανυστής

Στο κενό

$\vec{\nabla}\cdot\vec{E} = 0 \qquad (1)$

$\vec{\nabla}\cdot\vec{B} = 0 \qquad (2)$

$\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0 \qquad (3)$

$\vec{\nabla}\times\vec{B} - \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} = 0 \qquad (4)$

δυϊσμός $\vec{E} \rightarrow \vec{B}$

$\vec{B} \rightarrow -\mu_0\varepsilon_0\vec{E}$

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\kappa\lambda} F_{\kappa\lambda}$$

$\varepsilon^{\mu\nu\kappa\lambda}$ πλήρες αντισυμμετρικό σύμβολο (4^4)

$\varepsilon^{0123} = +1$

p = αριθμός μεταθέσεων μεταξύ δεικτών

$\varepsilon^{\mu\nu\kappa\lambda} = (-1)^p \varepsilon^{0123}$

$\varepsilon^{00\kappa\lambda} = -\varepsilon^{00\kappa\lambda} = 0$

όταν δύο δείκτες είναι ίσοι τότε τότε έχουμε 0

$\varepsilon^{0123} = +1 \qquad e^{1023} = -1\,\varepsilon^{0123} = -1 \qquad e^{1032} = -e^{1023} = -(-e^{0123}) = +1$

$\varepsilon_{0123} = -1 \qquad \varepsilon_{\mu\nu\kappa\lambda} = \eta_{\mu\mu'}\eta_{\nu\nu'}\eta_{\kappa\kappa'}\eta_{\lambda\lambda'}\varepsilon^{\mu'\nu'\kappa'\lambda'}$

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix} \qquad F^{\mu}{}_{\nu} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix} \qquad F_{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix}$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & E^T \\ -E & B \end{bmatrix} \qquad \eta^{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & I \end{pmatrix}$$

$$F^{\mu}{}_{\nu} = \eta_{\nu\kappa} F^{\mu\kappa} = F^{\mu\kappa}(\eta^T)_{\kappa\nu} = \mathbf{F}\cdot\mathbf{\eta} = \begin{bmatrix} 0 & E^T \\ -E & B \end{bmatrix}\begin{pmatrix} -1 & 0 \\ 0 & I \end{pmatrix} = \begin{bmatrix} 0 & E^T \\ E & B \end{bmatrix}$$

$$F_{\mu\nu} = \eta_{\mu\kappa} F^{\kappa}{}_{\nu} = \mathbf{\eta}\mathbf{F} = \begin{bmatrix} -1 & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} 0 & E^T \\ E & B \end{bmatrix} =$$

$$\tilde{F}^{00} = \frac{1}{2}\cancel{\varepsilon^{00\kappa\lambda}}^{\,0} F_{\kappa\lambda} = 0 \qquad \tilde{F}^{01} = \frac{1}{2}\varepsilon^{01\kappa\lambda}F_{\kappa\lambda} = \frac{1}{2}\left(\varepsilon^{0123}F_{23} + \varepsilon^{0132}F_{32}\right)$$

$\tilde{F}^{01} = F_{23}$ διοτι $\varepsilon^{0123} = -\varepsilon^{0132}$

$\tilde{F}^{01} = B_x \qquad F_{23} = -F_{32}$

$$\tilde{F}^{12} = \frac{1}{2}\varepsilon^{12\kappa\lambda}F_{\kappa\lambda} = \frac{1}{2}\varepsilon^{\kappa12\lambda}F_{\kappa\lambda} = \frac{1}{2}\varepsilon^{0123}F_{03} + \frac{1}{2}\varepsilon^{3120}F_{30} = F_{03}$$

$$\varepsilon^{12\kappa\lambda} = -\varepsilon^{1\kappa2\lambda} = \varepsilon^{\kappa12\lambda} \qquad \tilde{F}_{12} = -E_z$$

εξ. Maxwell στο κενο

οριζω $\partial_\mu \equiv \dfrac{\partial}{\partial x^\mu}$

συμπεριφορα συναλλοιωτη

$(\delta\eta\lambda\quad \partial_{\mu'} = \Lambda_{\mu'}{}^{\mu}\partial_\mu)$

$$\partial_\mu = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$\partial_\mu F^{\mu\nu} = 0$	$\partial_\mu \tilde{F}^{\mu\nu} = 0$
$\bar{\nabla}\cdot\bar{E} = 0$	$\bar{\nabla}\cdot\bar{B} = 0$
$\bar{\nabla}\times\bar{B} - \underbrace{\frac{1}{\mu_0\varepsilon_0}}_{1}\frac{\partial\bar{E}}{\partial t} = 0$	$\bar{\nabla}\times\bar{E} + \frac{\partial B}{\partial t} = 0$

$c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}} = 1$

$$\frac{\partial F^{\mu\nu}}{\partial x^\mu} = 0 \qquad \frac{\partial \tilde{F}^{\mu\nu}}{\partial x^\mu} = 0$$

$\nu=0 \quad \partial_\mu F^{\mu0} = \frac{\partial}{\partial t}(\cancel{F^{00}}) + \frac{\partial}{\partial x}F^{10} + \frac{\partial}{\partial y}F^{20} + \frac{\partial}{\partial z}F^{30} = -\bar{\nabla}\bar{E} = 0$

$\nu=1 \quad \partial_\mu F^{\mu1} = \frac{\partial}{\partial t}F^{01} + \frac{\partial}{\partial x}\cancel{F^{11}}^{\,0} + \frac{\partial}{\partial y}F^{21} + \frac{\partial}{\partial z}F^{31} =$

$$\frac{\partial E_x}{\partial t} + \frac{\partial}{\partial y}(-B_z) + \frac{\partial}{\partial z}(B_y) = \left[\frac{\partial E}{\partial t} - \bar{\nabla}\times\bar{B}\right]_x$$

$$\bar{\nabla}\times\bar{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} \qquad \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}$$

αναλλοίωτα του Η/Μ πεδίου

- $\frac{1}{2} F^{\mu\nu} F_{\mu\nu} = \frac{1}{2}\left(-\vec{E}\cdot\vec{E} + \vec{B}\cdot\vec{B} - \vec{E}\cdot\vec{E} + \vec{B}\cdot\vec{B}\right) = -\vec{E}\cdot\vec{E} + \vec{B}\cdot\vec{B} = -E^2 + B^2$
- $\frac{1}{4} \tilde{F}^{\mu\nu} F_{\mu\nu} = \frac{1}{4}\left(-\vec{E}\cdot\vec{B} - \vec{E}\cdot\vec{B} - \vec{E}\vec{B} - \vec{E}\cdot\vec{B}\right) = -\vec{E}\cdot\vec{B}$

Θεωρημα 6.5.1. (Χριστουδουλακη & Κορφιατη): εχουμε 2 ανεξαρτητα αναλλοιωτα

$F^{\mu}{}_{\mu} = 0 + 0 + 0 + 0$

$\eta_{\mu\nu} F^{\mu\nu} = F^{\mu}{}_{\mu}$

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}$$

$$\tilde{F}^{\mu\nu} = \begin{bmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z & E_y \\ -B_y & E_z & 0 & -E_x \\ -B_z & -E_y & E_x & 0 \end{bmatrix}$$

δυϊκος –

$$F_{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix}$$

- αν $E^2 > B^2$ E-τυπου
- αν $B^2 > E^2$ B-τυπου
- αν σε καποιο συστημα $\vec{B}=0$ τοτε $\vec{E}' \perp \vec{B}'$ διοτι $\vec{E}\cdot\vec{B} = 0$

$\bar{E}'_{\parallel} = \bar{E}_{\parallel}$ $\quad \bar{E}'_T = \gamma(\bar{E}_T + \bar{\upsilon}\times \cancel{\bar{B}_T}^{\,0}) = \gamma \bar{E}_T$

$\bar{B}'_{\parallel} = \cancel{\bar{B}_{\parallel}}$ $\quad \bar{B}'_T = \gamma(\cancel{\bar{B}_T}^{\,0} - \bar{\upsilon}\times\bar{E}_T)$

$\bar{B}' = \bar{B}'_{\parallel} + \bar{B}'_T = -\gamma\bar{\upsilon}\times\bar{E}_T = -\bar{\upsilon}\times\bar{E}'_T = -\bar{\upsilon}\times\bar{E}'$

$\bar{\upsilon}\times\bar{E}' = \bar{\upsilon}\times(\cancel{\bar{E}'_{\parallel}} + \bar{E}'_T)$

$$\boxed{\vec{B}' = -\bar{\upsilon}\times\vec{E}'}$$

- αν σε καποιο συστημα $\vec{E}=0$ τοτε

$$\boxed{\vec{E}' = \bar{\upsilon}\times\bar{B}'}$$

$\bar{\nabla}\cdot\bar{B} \longrightarrow \bar{\nabla}'\bar{B}'$

$\Lambda(\vec{v}=v\hat{x})$

$t = t(t',x') = \gamma(t'+vx')$
$x = x(t',x') = \gamma(x'+vt')$
$y = y'$
$z = z'$

$\bar{B}'_{\parallel} = \bar{B}_{\parallel} \qquad \bar{B}'_T = \gamma(\bar{B}_T + \bar{v}\times\bar{E})$

$(\bar{v}\times\bar{E} = \bar{v}\times\bar{E}_T)$

$$\bar{v}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v & 0 & 0 \\ E_x & E_y & E_z \end{vmatrix} = -vE_z\,\hat{y} + vE_y\,\hat{z}$$
$$\parallel$$
$$\bar{v}\times\bar{E}_T$$

$$\frac{\partial}{\partial x'} = \frac{\partial}{\partial x}\frac{\partial x}{\partial x'} + \frac{\partial}{\partial t}\frac{\partial t}{\partial x'} = \gamma\frac{\partial}{\partial x} + \gamma v\frac{\partial}{\partial t}$$

$$\frac{\partial}{\partial y'} = \frac{\partial}{\partial y} \qquad \frac{\partial}{\partial z'} = \frac{\partial}{\partial z}$$

$$B_x' = B_x \qquad B_y' = \gamma(B_y + vE_z) \qquad B_z' = \gamma(B_z - vE_y)$$

$$\bar{\nabla}'\bar{B}' = \left(\gamma\frac{\partial B_x'}{\partial x} + \gamma v\frac{\partial B_x'}{\partial t}\right) + \frac{\partial B_y'}{\partial y'} + \frac{\partial B_z'}{\partial z'} =$$

$$= \gamma\frac{\partial B_x}{\partial x} + \gamma v\frac{\partial B_x}{\partial t} + \frac{\partial}{\partial y}\left(\gamma(B_y + vE_z)\right) + \frac{\partial}{\partial z}\left(\gamma(B_z - vE_y)\right)$$

$$= \gamma\bar{\nabla}\bar{B} + \gamma v\left[\frac{\partial B_x}{\partial t} + \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right]$$

$$= \gamma\bar{\nabla}\bar{B} + \gamma v\left[\frac{\partial\bar{B}}{\partial t} + \bar{\nabla}\times\bar{E}\right]_x = 0$$

αληθής για κάθε $\vec{v}$

H/M (4η δοση)

ειδαμε οτι: $\bar{\nabla}' \bar{B}' = 0 \longrightarrow \left\{ \begin{array}{l} \bar{\nabla} \bar{B} = 0 \\ \dfrac{\partial \bar{B}}{\partial t} + \bar{\nabla} \times \bar{E} = 0 \end{array} \right\} \xrightarrow{\text{μπορουν να γραφουν ως}} \partial_\mu \tilde{F}^{\mu\nu} = 0$

"συναλλοιωτη" γραφη

δηλαδη $\partial_{\mu'} \tilde{F}^{\mu'\nu'} = \partial_\mu \tilde{F}^{\mu\nu}$

Αποδειξη:

$$\partial_{\mu'} \tilde{F}^{\mu'\nu'} = \Lambda_{\mu'}{}^{\kappa} \partial_\kappa \Lambda^{\mu'}{}_{\mu} \Lambda^{\nu'}{}_{\nu} \tilde{F}^{\mu\nu} = \Lambda^{\mu'}{}_{\mu} \Lambda_{\mu'}{}^{\kappa} \Lambda^{\nu'}{}_{\nu} \partial_\kappa \tilde{F}^{\mu\nu} = \Lambda^{\nu'}{}_{\nu} \partial_\mu \tilde{F}^{\mu\nu} = \Lambda^{\nu'}{}_{\nu} \left(\partial_\mu \tilde{F}^{\mu\nu} \right) = \emptyset$$

$$\delta^{\mu}_{\kappa} = \Lambda^{\mu'}{}_{\mu} \Lambda_{\mu'}{}^{\kappa} = \delta^{\kappa}_{\mu}$$

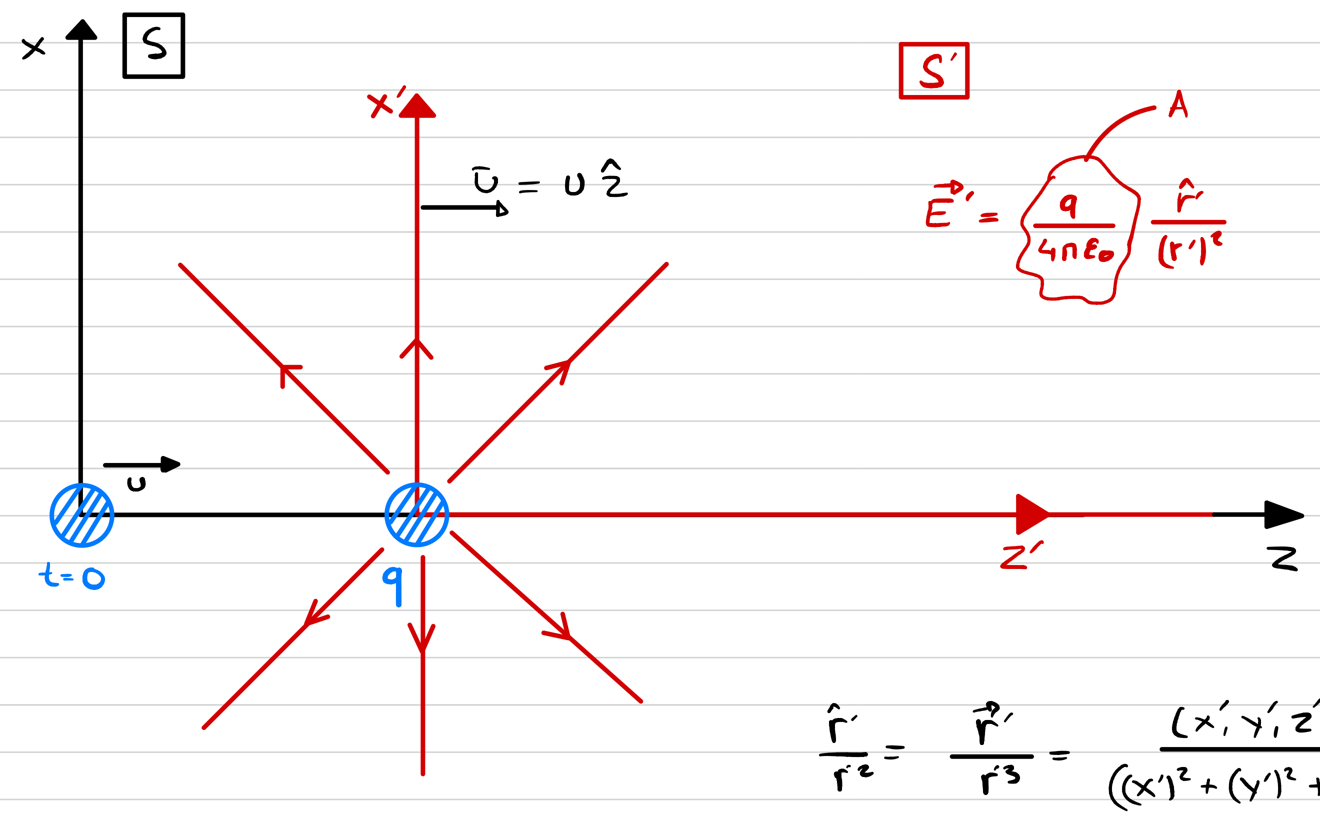
$$\Lambda_{\nu}{}^{\mu} = (\Lambda^{-1})^{\mu}{}_{\nu} = \eta_{\nu\kappa} \Lambda^{\kappa}{}_{\sigma} \eta^{\sigma\mu}$$

$\emptyset$ $\quad \forall \ \nu = 0,1,2,3$

$\partial_{\mu'} \tilde{F}^{\mu'\nu'} = \emptyset$ αν $\partial_\mu \tilde{F}^{\mu\nu} = \emptyset$

δηλαδη: οι εξ. Maxwell γραμμενες σε "συναλλοιωτη" μορφη δεν αλλαζουν υστερα απο μετασχηματισμους Lorentz

Η/Μ πεδίο κινουμενου φορτιου



$\boxed{S'}$

$$\vec{E}' = \underbrace{\frac{q}{4\pi\varepsilon_0}}_{A} \frac{\hat{r}'}{(r')^2}$$

$$\frac{\hat{r}'}{r'^2} = \frac{\vec{r}'}{r'^3} = \frac{(x', y', z')}{((x')^2 + (y')^2 + (z')^2)^{3/2}}$$

$$E_x = \gamma(E_x' - \bar{v}\times \cancel{B_T'}^{\,0}) = \gamma \frac{A\, x'}{((x')^2 + (y')^2 + (z')^2)^{3/2}}$$

$$E_y = \gamma(E_y' - \bar{v}\times \cancel{B_T'}^{\,0}) = \gamma \frac{A\, y'}{((x')^2 + (y')^2 + (z')^2)^{3/2}}$$

$$E_z = E_z' = A \frac{z'}{((x')^2 + (y')^2 + (z')^2)^{3/2}}$$

$z' = \gamma(z - vt)$

για $t = 0 = t'$

$z' = \gamma(z - vt)$

$x' = x$

$y' = y$

$$E_x\Big|_{t=0} = \gamma \frac{A\, x}{((x)^2 + (y)^2 + (\gamma z)^2)^{3/2}}$$

$$E_y\Big|_{t=0} = \gamma \frac{A\, y}{((x)^2 + (y)^2 + (\gamma z)^2)^{3/2}}$$

$$E_z\Big|_{t=0} = \gamma \frac{A\, z}{((x)^2 + (y)^2 + (\gamma z)^2)^{3/2}}$$

$$\vec{E} = \gamma \frac{A}{((x)^2 + (y)^2 + (\gamma z)^2)^{3/2}} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\boxed{\vec{E} = \frac{1}{\gamma^2(1 - v^2\sin^2\theta)^{3/2}} \frac{q}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2}}$$

$$((x)^2 + (y)^2 + (\gamma z)^2)^{3/2} = \left[r^2\sin^2\theta + \gamma^2 r^2\cos^2\theta\right]^{3/2} = r^3\left(\sin^2\theta + \gamma^2(1 - \sin^2\theta)\right)^{3/2}$$

$$= r^3(\gamma^2 - \gamma^2 v^2 \sin^2\theta)^{3/2}$$

$$r^3\gamma^3(1 - v^2\sin^2\theta)^{3/2}$$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$\theta = 0 \qquad \vec{E} = \dfrac{1}{\gamma^2} A \dfrac{\hat{r}}{r^2}$

$\theta = \dfrac{\pi}{2} \qquad \vec{E} = A\dfrac{\hat{r}}{r^2}$

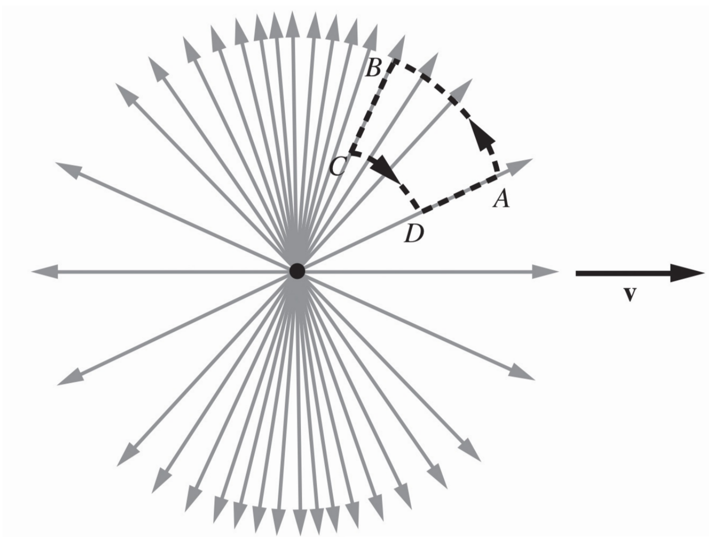
$\bar{B} = \bar{v}\times\bar{E}$ όταν $\bar{B}' = 0$

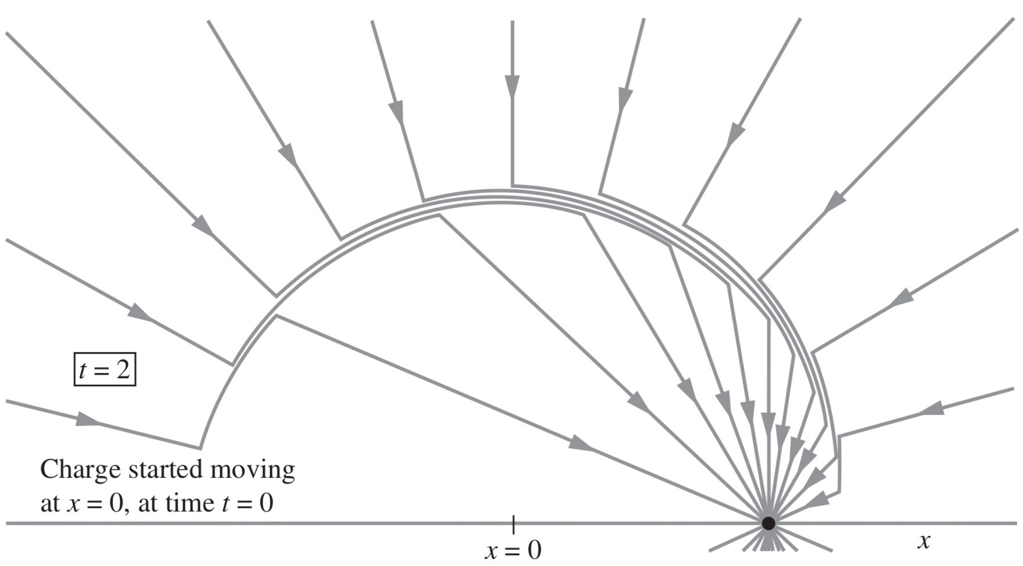
$v \cdot E \cdot (\hat{z}\times\hat{r})$

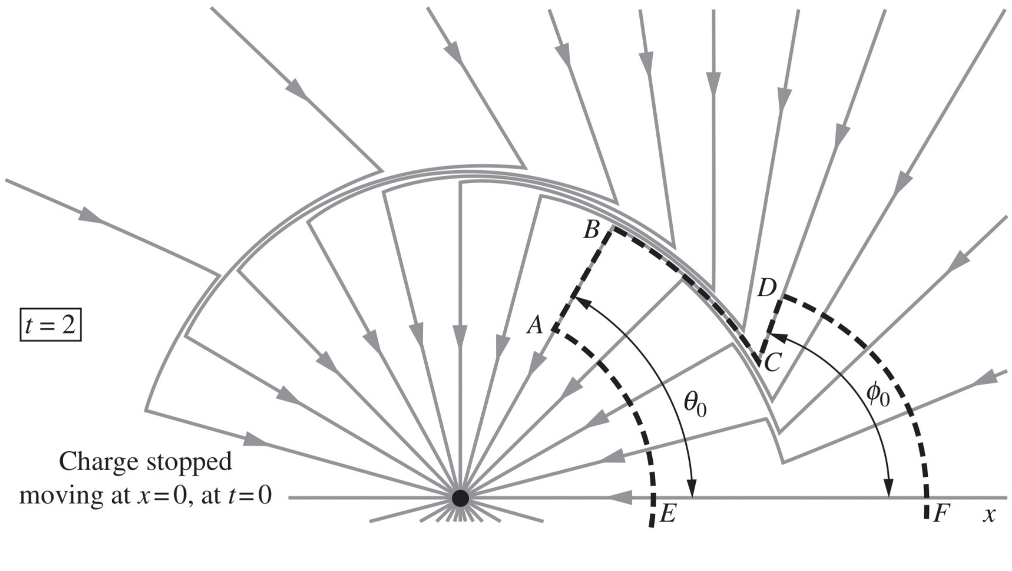
↳ $\sin\theta\,\hat{\phi}$

$$\boxed{\bar{B} = \frac{v\sin\theta}{\gamma^2(1 - v^2\sin^2\theta)^{3/2}} \frac{q}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2}\,\hat{\phi}}$$

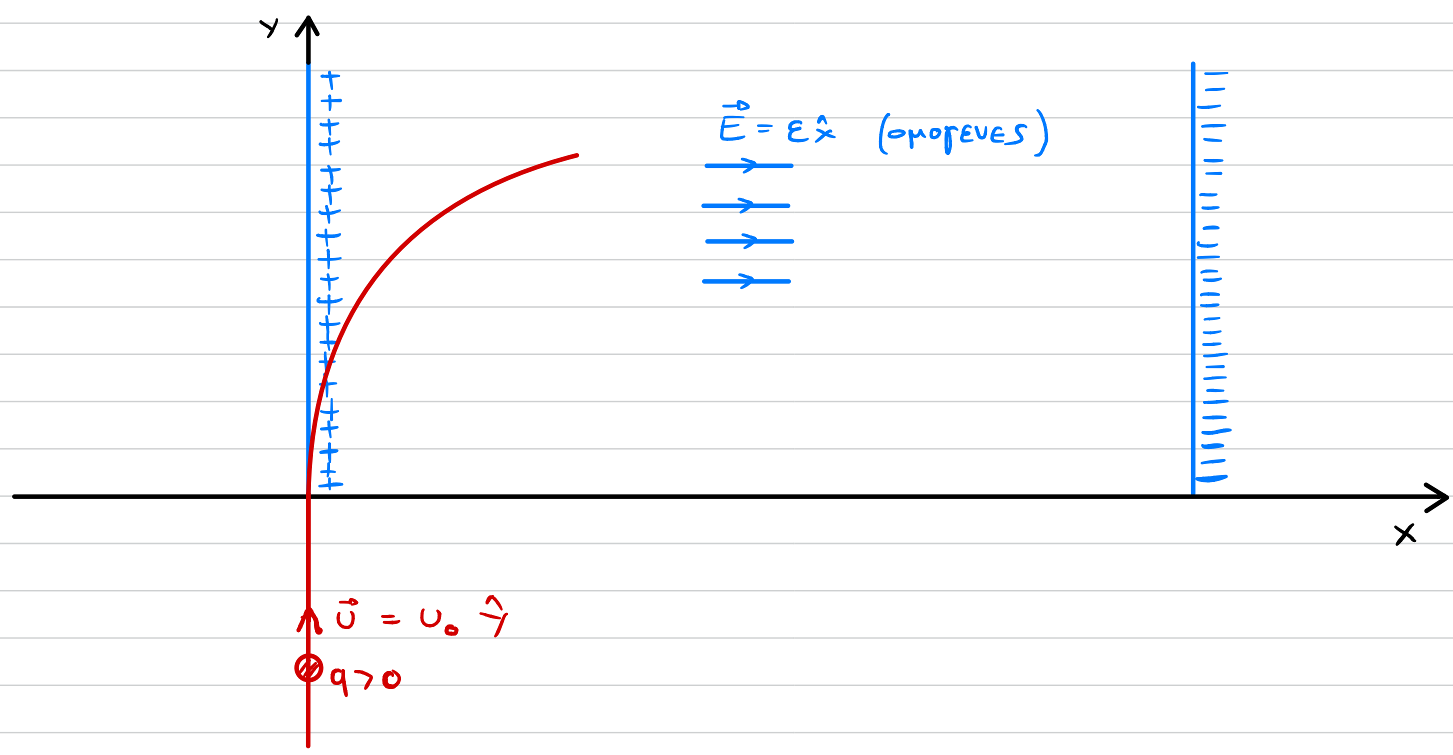
Electricity & Magnetism
Purcell M. Edward







κινηση φορτιου σε σταθερο Ε-πεδιο



$$\vec{F}_L = q(\vec{E} + \vec{u}\times\vec{B}) = q\mathcal{E}\hat{x} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(\gamma m \vec{u})$$

$x(t)$, $y(t)$ = ?
$u_x(t)$, $u_y(t)$ = ?

A:	$u_y(t)$ = σταθ	46%
B:	$u_y(t)$ = αυξουσα	15%
C:	$u_y(t)$ = φθινουσα	39%

$$F^\mu = q F^{\mu\nu} u_\nu$$

$$\frac{dP^\mu}{d\tau} = \gamma \begin{bmatrix} dE/dt \\ dP_x/dt \\ dP_y/dt \\ dP_z/dt \end{bmatrix} = q \begin{bmatrix} 0 & \mathcal{E} & 0 & 0 \\ -\mathcal{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \gamma \begin{bmatrix} -1 \\ u_x \\ u_y \\ u_z \end{bmatrix}$$

$$dE/dt = q\mathcal{E}u_x = (q\mathcal{E}\hat{x})\cdot(u_x\hat{x}) = \vec{F}\cdot\vec{u}$$

$$\dot{P}_x = q\mathcal{E} \rightsquigarrow P_x = q\mathcal{E}t + \cancel{\text{σταθ}}\ 0$$

$$\dot{P}_y = 0 \rightsquigarrow P_y = \text{σταθ} = \frac{m u_0}{\sqrt{1-u_0^2}} = \frac{m\, u_y(t)}{\sqrt{1-u^2}} = P_{y0}$$

$$\dot{P}_z = 0 \rightsquigarrow P_z = \text{σταθ} = 0$$

$$u_x(t) = \frac{dx}{dt} = \frac{P_x}{E} = \frac{q\mathcal{E}t}{\sqrt{m^2 + P_{y0}^2 + P_x^2}} = \frac{q\mathcal{E}t}{\sqrt{m^2 + P_{y0}^2 + (q\mathcal{E}t)^2}} = \frac{q\mathcal{E}t}{\sqrt{E_0^2 + (q\mathcal{E}t)^2}}$$

$$= \frac{q\mathcal{E}t/E_0}{\sqrt{1 + \left(\frac{q\mathcal{E}}{E_0}t\right)^2}}$$

$$E_0^2 \equiv m^2 + P_{y0}^2$$

$$A \equiv \frac{q\mathcal{E}}{E_0}$$

$$dx = \frac{1}{A}\int \frac{At}{\sqrt{1 + (At)^2}}\, d(At)$$

$$x(t) = \frac{1}{A}\sqrt{1+(At)^2} + \sigma\tau\alpha\theta \quad \Big| \quad x(0) = 0 \;\rightarrow\; \sigma\tau\alpha\theta = -\frac{1}{A}$$

$$x(t) = \frac{1}{A}\left(\sqrt{1+(At)^2} - 1\right)$$

εχουμε $P_y(t) = P_{y0} = \sigma\tau\alpha\theta$

$$dy/dt = \frac{P_y(t)}{E} = \frac{P_{y0}}{\sqrt{m^2 + P_{y0}^2 + P_x^2}}$$

$$= \frac{P_{y0}}{\sqrt{E_0^2 + (q\mathcal{E}t)^2}} = \frac{P_{y0}/E_0}{\sqrt{1+(At)^2}} = u_y(t)$$

$$dy = \int \frac{P_{y0}/E_0}{\sqrt{1+(At)^2}}\, dt$$

$$\cosh^2\omega = 1 + \sinh^2\omega$$

$$\sinh\omega = At$$

$$\frac{d\omega}{A}\cosh\omega = dt$$

$$\int dy = P_{y0}/E_0 \int \frac{\cosh\omega}{\sqrt{1+\sinh^2\omega}}\,\frac{d\omega}{A}$$

$$= \frac{P_{y0}}{AE_0}\int d\omega = \frac{P_{y0}}{AE_0}\,\omega + \cancel{\sigma\tau\alpha\theta} \rightarrow 0 \qquad y(0) = 0$$

$$y(t) = \frac{P_{y0}}{AE_0}\sinh^{-1}(At)$$

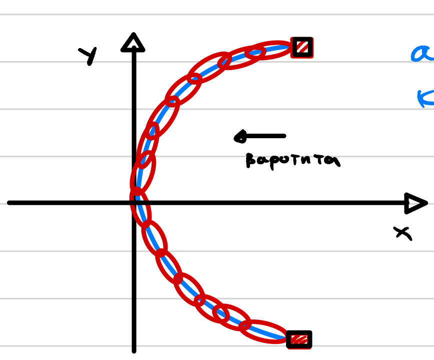
$$x(t) = \frac{E_0}{q\mathcal{E}}\left(\sqrt{1+\left(\frac{q\mathcal{E}t}{E_0}\right)^2} - 1\right) \qquad y(t) = \frac{P_{y0}}{q\mathcal{E}}\sinh^{-1}\left(\frac{q\mathcal{E}t}{E_0}\right)$$

$x(y)$

$$\sinh\left(\frac{q\mathcal{E}\cdot y}{P_{y0}}\right) = \frac{q\mathcal{E}t}{E_0}$$

$$x(y) = \frac{E_0}{q\mathcal{E}}\left(\sqrt{1+\left(\sinh\left(\frac{q\mathcal{E}\cdot y}{P_{y0}}\right)\right)^2} - 1\right)$$

$$x(y) = \frac{E_0}{q\mathcal{E}}\left[\cosh\left(\frac{q\mathcal{E}\cdot y}{P_{y0}}\right) - 1\right]$$



αλυσοειδης καμπυλη

$y(x) \sim \frac{1}{a}\cosh(ax)$